Due date: October 28, 2011

Read Chapters 4 and 5 of the textbook.

1. Problem 4.4.
2. Problem 4.7.
3. (a) Problem 5.3
   (b) Problem 5.6
4. (*) The following problem is called the \textit{joint replenishment problem}. There is a discrete time horizon $t = 1, \ldots, T$, and for each such time there is a given (non-negative integer) demand $d_{it}$ for each commodity $i = 1, \ldots, k$. An order at time $t$ can satisfy the demand at any time $t$ or later. There is a fixed cost $K_0$ for placing an order (for any number of units of any number of commodities) at each time $t = 1, \ldots, T$. In addition, for each commodity $i = 1, \ldots, k$, there is a fixed cost $K_i$ for placing an order (for any number of units of commodity $i$) at each time $t = 1, \ldots, T$. Furthermore, for each commodity $i = 1, \ldots, k$, and for each pair of times $(s, t)$, where $s < t$, there is a per-unit holding cost $h_{st}$ for keeping commodity $i$ in inventory from being ordered at time $s$ to meeting demand at time $t$. (The demand at time $t$ for commodity $i$ need not be satisfied by orders placed at a unique time.) The aim is to minimize the total ordering plus holding cost incurred. (This should remind you in some ways of the uncapacited facility location problem.) Give an integer programming formulation of this problem (in which the decision variables should indicate in which time periods there are orders placed, in which time periods there are orders of commodity $i$ placed, and the fraction of the demand for each commodity at each time that is served from a given potential order point). Use the LP relaxation of this formulation to derive a constant approximation algorithm for the joint replenishment problem. (It can be deterministic or randomized.) Try to keep the constant reasonable; 4 is doable without too much difficulty.