Problem #1

The goal for this version of the cutting stock problem is to implement a version of the previous column generation scheme. However, in this instance, we have the initial problem of devising an initial basic feasible solution. Consider the following algorithm:

Step 1: Start with the initial variable set $x_1, \ldots, x_m$ with $x_i$ corresponding to a roll configuration of one unit of item $i$.

Step 2: Note that the above variables will form an initial basis for the problem $\min \{\sum_j x_j : \sum_j a_{ij} x_j \geq .9r_i, 1 \leq i \leq m, x \geq 0\}$. Run the previous column generation scheme on this problem until we reach solution with objective value no larger than $N$. If we reach optimality and have optimal objective function value $> N$, STOP: Original problem is infeasible.

Step 3: Let $x$ be the solution above. Let $I$ be the set of $i$ such that the constraint $\sum_j a_{ij} x_j \leq 1.1r_i$ holds. Iteratively choose some $i$ for which it is not satisfied, and apply the column generation scheme to the LP $\min \{\sum_j a_{ij} x_j : \sum_j a_{kj} x_j \geq .9r_k \ \forall k, \sum_j a_{kj} x_j \leq 1.1r_k \ \forall k \in I, \sum_j x_j \leq N, x \geq 0\}$. If the optimal solution has objective value $> 1.1r_i$, stop: problem infeasible. Else, add $i$ to $I$ and repeat until $I = \{1, \ldots, m\}$.

Step 4: At this point, you have a basic feasible solution to the original problem, hence, you can finish solving the problem via the original column generation scheme.

Problem #2

Suppose we are given the Lagrange multipliers, $\lambda$. Consider relaxing the machine constraints. This gives us a relaxation of the form:

$$\min \ \sum_{ij} (c_{ij} + \lambda p_{ij}) x_{ij} - \sum_i M_i$$

s.t. $\sum_i x_{ij} = 1 \ \forall j$

$x_{ij} \geq 0$

Note that these constraints are now independent of each other. Hence, we
can quickly find an optimal solution to the above LP as follows: for each $j$, let $i_j = \arg\min_i \{(c_{ij} + \lambda p_{ij})\}$. Set $x_{i_i, j} = 1$ for $j = 1, \ldots, n$ and $x_{ij} = 0$ otherwise.