11.1 Overview of the last lecture

Recall setting from last lecture of a 2-sided platform with symmetric sides L and R.

Each user $i$ in side L arrives with 2 parameters $(b^L_i, B^L_i) \sim F^L(b_i, B_i)$ where $b$ is their interaction benefit, $B$ is their membership benefit, and $F^L$ is a distribution with density $f^L(b_i, B_i)$.

User $i$ gains utility $u^L_i$ where:

$$u^L_i = b^L_i N^R + B^L_i - P^L(N^R)$$

and will only enter the platform if this $u^L_i \geq 0$.

Last lecture we showed that:

$$N^L = \int_{-\infty}^\infty \int_{P^L(N^R) - b^L_i N^R}^\infty f^L(b^L, B^L) dB^L db^L$$

and

$$V^L = \int_{-\infty}^\infty \int_{P^L(N^R) - b^L_i N^R}^\infty (b^L_i N^R + B^L_i) f^L(b^L, B^L) dB^L db^L$$

where $N^L$ is the number of users in side L and $V^L$ is the welfare of users in side L.

11.2 Overview of this lecture

Assume $f$ is nonzero everywhere. Then increasing $P^L(N^R)$ always decreases the number of users in side L. Furthermore, if we fix some target $\hat{N}^L$ then there is a unique price $P^L(\hat{N}^L, N^R)$ that will result in $\hat{N}^L$ users. We call this $P^L(\hat{N}^L, N^R)$ an insulating tariff.

Typically, $P^L(N^R)$ will look like:

$$P^L(N^R) = P^L + p^L N^R$$

where $P^L$ is a subscription fee and $p^L$ is a transaction fee.

Today we will look at how to pick $P^L$ to maximize welfare and revenue.
11.3 Welfare and Profit Maximization

11.3.1 First order conditions

Welfare = \( W(N^L, N^R) = V^L + V^R - c^L N^L - c^R N^R - cN^L N^R \)

Profit = \( \Pi(N^L, N^R) = P^L N^L + P^R N^R - c^L N^L - c^R N^R - cN^L N^R \)

where \( c^L, c^R, \) and \( c \) are marginal costs incurred to run and maintain the platform.

In a ride-sharing platform \( c^{riders} \) may be marketing costs, \( c^{drivers} \) the cost of onboarding a new driver, and \( c \) the cost of insuring each ride.

The first order condition for welfare maximization is

\[
\frac{\partial W}{\partial N^L} = 0 \implies \frac{\partial V^L}{\partial N^L} + \frac{\partial V^R}{\partial N^L} = c^L + cN^R
\]

and the first order condition for profit maximization is

\[
\frac{\partial \Pi}{\partial N^L} = 0 \implies P^L + N^L \frac{\partial P^L}{\partial N^L} + N^R \frac{\partial P^R}{\partial N^L} = c^L + cN^R
\]

11.3.2 Solving for partial derivatives via Leibniz’s Rule

1. Recall last lecture we solved for

\[
\frac{\partial N^L}{\partial N^L} = 1 \implies \frac{\partial P^L}{\partial N^L} = \int_{-\infty}^{\infty} f^L(b^L, P^L - b^L N^R) db^L
\]
We can interpret the integral in the denominator as the mass of people sitting on the
margin. The interpretation here is that by perturbing the price of side L up slightly,
you lose exactly those people who had 0 utility.

2. We also computed
\[ \frac{\partial N_L}{\partial N_R} = 0 = \frac{\partial P_L}{\partial N_R} = \frac{\int_{-\infty}^{\infty} b_L f_L(b_L, P_L - b_L N_R) db_L}{\int_{-\infty}^{\infty} f_L(b_L, P_L - b_L N_R) db_L} = \tilde{b}_L(N_L, N_R) \]

where \( \tilde{b}_L(N_L, N_R) \) is the AIVMU (average interaction value of marginal users)

3. Applying Leibniz’s rule on \( V^L \) yields
\[ \frac{\partial V^L}{\partial N_L} = -\int_{-\infty}^{\infty} P^L f^L(b_L, P_L - N_R b_L) \frac{\partial P_L}{\partial N_L} db_L = P^L \]
This matches our intuition since the marginal users have utility 0.

4.
\[ \frac{\partial V^L}{\partial N_R} = -\int_{-\infty}^{\infty} P^L f^L(b_L, P_L - N_R b_L) \frac{\partial P_L}{\partial N_R} db_L + \int_{-\infty}^{\infty} \int_{P_L - b_L N_R}^{\infty} b_L f^L(b_L, B^L) dB^L db_L \]
\[ = 0 + \int_{-\infty}^{\infty} \int_{P_L - b_L N_R}^{\infty} b_L f^L(b_L, B^L) dB^L db_L \]
\[ = N_L \tilde{b}_L \]
This is the expected interaction benefit \( b_L \) over all users in side L and is called the AIVU (average interaction value of user). Intuitively, each user gains \( \tilde{b}_L \) in interaction benefit whenever someone joins the other side of the platform.

11.3.3 Welfare
Substituting the partial derivatives we computed into the first order condition gives:
\[ P^L_W = c^L + c N_R - N_R \tilde{b}_R \]
\( c^L + c N_R \) is the marginal cost of adding a user to side L. \( N_R \tilde{b}_R \) can be interpreted as a user’s externality, the benefit given to the other side of the platform.
11.3.4 Profit

\[ P_L + N^L \frac{\partial P_L}{\partial N^L} = c^L + cN^R - N^R \tilde{b}^R \]

Define **price elasticity of demand** as

\[ \eta^L(N^L, N^R) = -\frac{\partial N^L/N^L}{\partial P_L/P_L} \]

Typically price elasticity is positive since as price increases, customers decrease.

Now we can rewrite

\[ P_L = c^L + cN^R + \frac{P_L}{\eta^L} - N^R \tilde{b}^R \]

\[ \mu^L = \frac{P_L}{\eta^L} \]

is called the **Cournot distribution** or **market power**.

We can also look at the difference between these two prices:

\[ P_L - P_W = \mu^L + N^R(\tilde{b}^R - \bar{b}^R) \]

11.3.5 Examples

**Rouchet-Tirole (2003)**

Assume \( B^L, B^R, c^L, c^R = 0 \), i.e. no membership benefits or costs. Then

\[ N^L(N^R, P^L) = 1 - F^L\left(\frac{P^L}{N^R}\right) \]

\[ P^L(N^R, \tilde{N}^L) = (F^L)^{-1}(1 - \tilde{N}^L)N^R \]

The only fee is a per transaction fee \( p^L = (F^L)^{-1}(1 - \tilde{N}^L)N^R \).


Assume \( c = 0 \) and \( b^L, b^R \) fixed. Then

\[ N^L = 1 - F^L(P^L - b^L N^R) \]

\[ P^L(N^R, \tilde{N}^L) = (F^L)^{-1}(1 - \tilde{N}^L) + b^L N^R \]

**Scaled income model**

\[ u_i^L = b_i^L (N^R + \frac{1}{\beta^L}) \] where \( \beta^L = \frac{b^L}{b^L} \)