The pricing problem

- Single good, single buyer with value \( V \sim F \)
- Can set price \( P \), buyer buys if \( V \geq P \)

\[
R^* = \max_{P \geq 0} P \cdot (1 - F(P)) = \max_{q \in [0,1]} F^{-1}(1-q) \cdot q
\]

Myerson price \( P^* : \quad P^* - \frac{1 - F(P^*)}{f(P^*)} = 0 \)

\( F \) is regular iff \( P - \left( \frac{1 - F(P)}{f(P)} \right) \) non-decreasing in \( P \)

- To go beyond this, we need to define 3 things:
  1) Behavioral model of buyer
  2) Objectives and constraints of seller
  3) Structure of available information
The rest of the course considered several such settings:

- **Buyer model** - perfect segmentation
  - probabilistic choice
  - strategic behavior

- **Seller models** - limited capacity
  - admission control (five classes)
  - network externalities
  - dynamic pricing
  - DSIC mechanisms
  - 2-sided marketplace platforms

- **Information structure** - full knowledge of buyer value/choice distributions
  - learning from data (spiral-down!)
  - no knowledge (DSIC mechanisms)
    - Bubow-Kempner
Main ideas and techniques

1) DP formulation for pricing problems (value function, Bellman equation, optimal control)

2) Protection level policies (for single-resource allocation)
   (2.5) Convexity, Jensen’s inequality

3) Fluid approximations for complex DPs (and the bid-price heuristic)

4) The spiral-down effect (importance of using the correct model, effect of improper learning)

5) Probabilistic choice models for buyer behavior: Luce’s axioms and the MNL
6) Assortment optimization under the MNL model - optimality of nested-by-revenue sets

7) Mechanism design - The Vickrey auction, dominant strategy incentive compatibility (DSIC).

8) Myerson's Lemma - DSIC $\iff$ monotone allocation rule (for single parameter settings)

9) Optimal revenue DSIC mechanism $\Rightarrow$ maximize "virtual welfare"
   (reserve prices, Bulow-Klemperer theorem)

10) 2-sided marketplace optimization - choose insulating prices $P^L(N^L, N^R), P^R(N^L, N^R)$; optimize over $N^L, N^R$
Beyond single parameters settings - things get strange.

E.g. - single buyers, 2 non-identical items
- Values $v_1, v_2 \sim F$ iid, additive utilities

1) $v_1, v_2 \sim \begin{cases} 1 & \text{wp 0.5} \\ 2 & \text{wp 0.5} \end{cases}$

- Sell both separately $\Rightarrow R = 2$ (for $p = 1$ or 2)
- Sell bundle at price 3 $\Rightarrow R = 3 \cdot (1 - \frac{1}{4}) = \frac{9}{4}$

2) $v_1, v_2 \sim \begin{cases} 0 & \text{wp } \frac{1}{3} \\ 1 & \text{wp } \frac{1}{3} \\ 2 & \text{wp } \frac{1}{3} \end{cases}$

- Sell separately (at $p = 1$ or 2) $\Rightarrow R = \frac{4}{3}$
- Sell bundle (at $p = \frac{2}{3}$) $\Rightarrow R = \frac{4}{3}$

$R_{\text{offer 1 item at } p = 2 \text{ or bundle at } p = 3} = \frac{13}{9} > \frac{4}{3}$