Models of customer behavior

- Suppose we want to sell \( m \) different items among \( n \) buyers - how do buyers choose items?

3 models of customer behavior

i) Perfect segmentation - Each customer only wants a single item from the set of items

ii) (Probabilistic) Choice model - Each customer chooses an item from amongst the displayed items

iii) Strategic choice - Customers compete with each other to try and get the "best deal" for themselves.
Auctions and mechanism design

- Up till now, we looked at perfect segmentation and probabilistic choice, and used pricing, capacity control and assortment control as our optimization tools.

- We now introduce a model for strategic customers, and a new optimization tool - auctions.

- Consider a setting where we want to sell 1 item.

  Quasilinear utility model

  - Each bidder $i$ has an independent value $v_i$ for the item. This value is private

  - If the bidder is offered the item at price $p \leq v_i$, then its utility is $v_i - p$

  - If the bidder is not offered the item (or offered at price $p > v_i$), then its utility is 0
Sealed-bid auctions

These occur in three steps:

i) (Bidding) Each bidder \( i \) communicates bid \( b_i \) to seller.

ii) (Allocation Rule) Seller chooses bidder who gets the item (if anyone).

iii) (Payment Rule) Seller decides on price.

- Natural allocation rule - sell to highest bidder.
- Payment rule? This affects bidder behavior!

Eg - What if price = 0?

Then everyone tries to set \( b_i \) as high as possible!
* First-price auctions

- Set payment \( P = \max_i [b_i] \)
  Allocate item to \( i^* = \arg \max_i [b_i] \)
- Problem: Very difficult for bidders to decide their bid!

* Second-price auction (Vickrey auction)

- Allocate item to \( i^* = \arg \max_i [b_i] \)
  Set payment \( P = \max_{i 
eq i^*} \max_j [b_{ij}] \)
- This is equivalent to an ascending price auction
- Now what should bidder i bid?
We now show two properties of the Vickrey auction:

i) In the Vickrey auction, every bidder $i$ sets her bid $b_i = \text{private valuation } \theta_i$, no matter what the other bidders do (dominant strategy).

**Pf:** Let $b_{-i} = \text{vector of bids of all bidders other than } i$.

- Fix some arbitrary bidder $i$, valuation $\theta_i$, bids $b_i$.
- Let $B = \max_{j \neq i} b_j$, and suppose $i$ knows $B$.

- There are 2 cases:
  i) If $\theta_i < B$, then bidder $i$ can get a utility of $\max \{ 0, \theta_i - B \} = 0$, which can be achieved by setting $b_i = \theta_i$.
  ii) If $\theta_i \geq B$, then bidder $i$ can get utility of $\max \{ 0, \theta_i - B \} = \theta_i - B$, again by setting $b_i = \theta_i$. 


2) In the Vickrey auction, every truth-telling bidder has non-negative utility.

\[ \text{Pf} \ - \ \text{If bidder } i \text{ loses, then utility } = 0 \]

\[ \text{- If bidder } i \text{ wins (while bidding } b_i = v_i), \text{ then utility is } v_i - B \geq v_i - b_i = 0 \] (as } b_i \geq B \]

Henceforth, we want all mechanisms to have these 2 properties:

1. (Dominant Strategy) Incentive Compatibility (DSIC)
   - Bidder i's utility is maximized by setting } b_i = v_i, \text{ no matter what other bidders bid.}
2. Individual Rationality (IR)
   - Every bidder has non-negative utility assuming truth-telling.

As shorthand, we will call a mechanism with these two properties to be DSIC. Our aim is to design DSIC mechanisms to maximize some given objective.