

Lecture 9

The Hungarian Algorithm

0. Set up a cost table. Assume all costs are ≥ 0 .
 1. For each row $i = 1, \dots, n$
 let α_i = the smallest entry in row i
 update: $t_{ij} \leftarrow t_{ij} - \alpha_i$ for all entries (i,j) in row i
 2. For each col $j = 1, \dots, n$
 let β_j = the smallest entry in column j
 update: $t_{ij} \leftarrow t_{ij} - \beta_j$ for all entries (i,j) in col j
 3. Check if there is an all-zero assignment
 If there is one, this assignment is optimal
 If there isn't one:
 - find a zero-cover containing fewer than n lines,
 - let δ = the smallest entry not covered by any lines
 - update:
 - $t_{ij} \leftarrow t_{ij} - \delta$ if entry (i, j) is not covered by any lines
 - $t_{ij} \leftarrow t_{ij}$ if entry (i, j) is covered by exactly one line
 - $t_{ij} \leftarrow t_{ij} + \delta$ if entry (i, j) is covered by exactly two lines
- (Repeat step 3 until there is an all-zero assignment)

Wrapping up Network Optimization

- Models/Problems:
 - Minimum-cost network flow (“mincost flow”)
 - Maximum flow (“maxflow”)
 - The shortest path problem
 - The assignment problem
 - The longest-path problem
 - Maximum bipartite matching
 - Bipartite minimum vertex-cover
 - Lots more!

Wrapping up Network Optimization

- Algorithms:
 - The Ford-Fulkerson Algorithm
 - The Hungarian Algorithm
 - Simplex method (when network problems are written as linear programs)

Wrapping up Network Optimization

- Theorems and other results
 - The integrality theorem for mincost flow
 - The integrality theorem for maxflow
 - The maxflow/mincut theorem
 - ...

Wrapping up Network Optimization

- Other new stuff in your “Optimizer’s Purse”:
 - Modeling and solving using AMPL
 - How to formulate a problem as a given model (the 3-step framework!)
 - Formulating network optimization problems as LPs
 - Formulating various problems as mincost flow
 - Formulating various problems as maxflow
 - Utilizing LP duality to analyze network problems

Prelim 1: Thursday April 21, 7:30pm – 9pm (1.5 hrs), Kimball B11

- All network optimization topics, including the assignment problem and the Hungarian algorithm and LP duality.
- The amount of AMPL in the exam will be little if any.
- Recitations this week will consist of review and overview of practice prelim solutions.
- Types of questions will have similar flavors as homework problems.
- Check for **prelim conflicts** and let me know ASAP.

Next time in Opt 2:
On to Dynamic Programming!

i>clicker question

(after the break)

Q1: Consider our example

What is the shortest s-K path found using our DP method so far?

A. $s \rightarrow A \rightarrow G \rightarrow K$

B. $s \rightarrow B \rightarrow F \rightarrow K$

C. $s \rightarrow C \rightarrow F \rightarrow K$

D. $s \rightarrow D \rightarrow H \rightarrow K$

E. The information available is not sufficient to determine the shortest s-K path.

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