

Lecture 26

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Recap from last lecture

2. Constrained Optimization

$$\begin{array}{ll} \min & f(x) \\ s.t. & g_i(x) \geq 0 \quad \forall i \in \{1, \dots, p\} \quad (NLP) \\ & h_j(x) = 0 \quad \forall j \in \{1, \dots, q\} \end{array}$$

2. Constrained optimization

Necessary and sufficient condition of optimality

Suppose f, g_i, h_j are continuous, differentiable.

If \bar{x} is an optimal solution for (NLP), then there exists $y_i \geq 0, z_j$ that satisfy :

- If $g_i(\bar{x}) > 0$, then $y_i = 0$,

- and $\nabla L_{y,z}(\bar{x}) = 0$

2. Constrained optimization

Necessary and sufficient condition of optimality

Suppose f, g_i, h_j are continuous, differentiable.

If \bar{x} is an optimal solution for (NLP), then there exists $y_i \geq 0, z_j$ that satisfy :

- If $g_i(\bar{x}) > 0$, then $y_i = 0$,

- and $\nabla f - \sum_{i=1}^p y_i g_i(\bar{x}) - \sum_{j=1}^q z_j h_j(\bar{x}) = 0$

Example:

Nonlinear constrained optimization

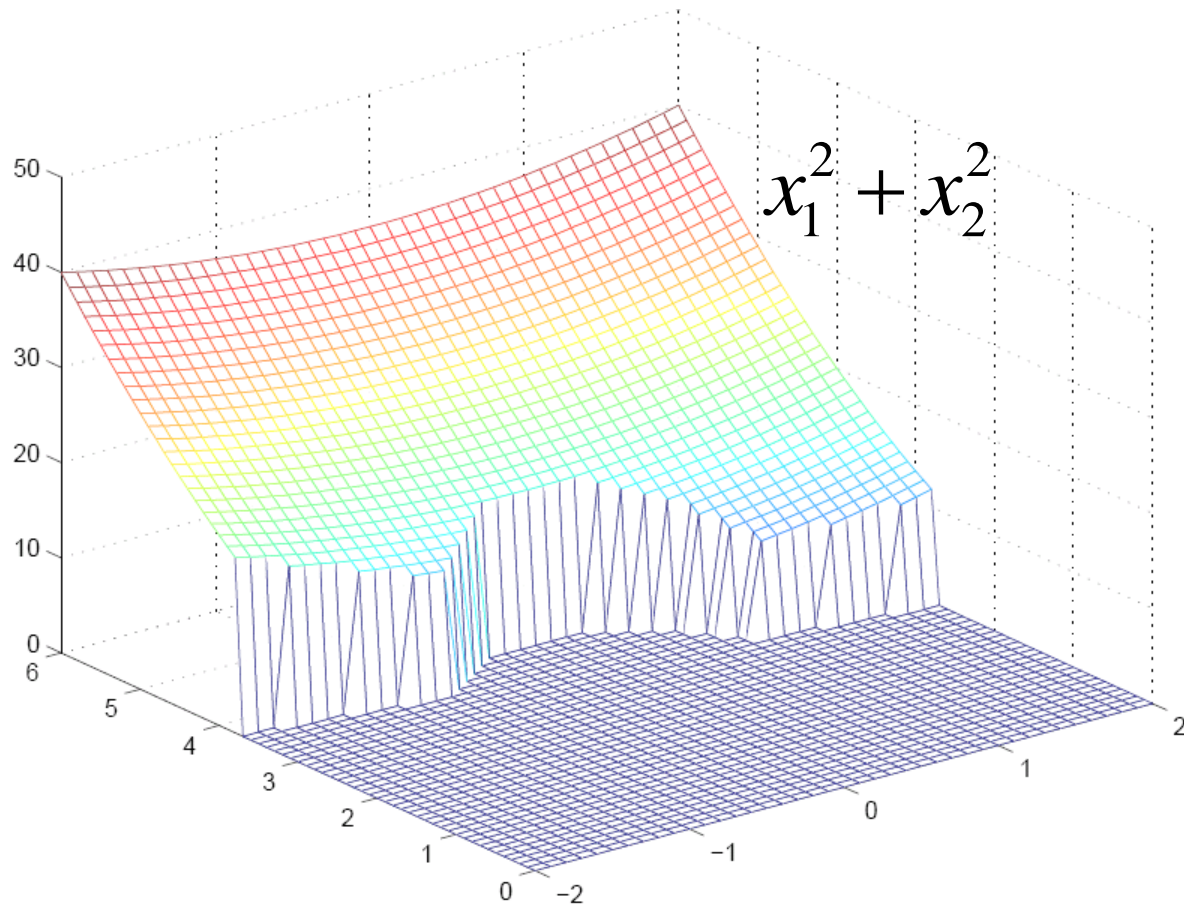
$$\min \quad x_1^2 + x_2^2$$

$$s.t. \quad x_1^2 + x_2 \geq 4$$

$$x_1 + 4x_2 \geq 13$$

Example:

Nonlinear constrained optimization



Example:

Nonlinear constrained optimization

$$\min \quad x_1^2 + x_2^2$$

$$s.t. \quad x_1^2 + x_2 \geq 4$$

$$x_1 + 4x_2 \geq 13$$

Example:

Nonlinear constrained optimization

$$\min \quad f(x)$$

$$s.t. \quad g_1(x) \geq 0$$

$$g_2(x) \geq 0$$

$$f(x) = x_1^2 + x_2^2$$

$$g_1(x) = x_1^2 + x_2 - 4$$

$$g_2(x) = x_1 + 4x_2 - 13$$

Example:

Nonlinear constrained optimization

$$L_y(x) = f(x) - y_1 g_1(x) - y_2 g_2(x)$$

$$\nabla L_y(x) = \nabla f(x) - y_1 \nabla g_1(x) - y_2 \nabla g_2(x)$$

$$f(x) = x_1^2 + x_2^2$$

$$g_1(x) = x_1^2 + x_2 - 4$$

$$g_2(x) = x_1 + 4x_2 - 13$$

Q: Does the point $x=(10, 20.25)$ satisfy the necessary optimality conditions?

$$\min \quad f(x)$$

$$s.t. \quad g_1(x) \geq 0$$

$$g_2(x) \geq 0$$

$$f(x) = x_1^2 + x_2^2$$

$$g_1(x) = x_1^2 + x_2 - 4$$

$$g_2(x) = x_1 + 4x_2 - 13$$

2. Constrained optimization

Necessary and sufficient condition of optimality

Suppose f, g_i, h_j are continuous, differentiable.

If \bar{x} is an optimal solution for (NLP), then there exists $y_i \geq 0, z_j$ that satisfy :

- If $g_i(\bar{x}) > 0$, then $y_i = 0$,

- and $\nabla L_{y,z}(\bar{x}) = 0$

i>clicker

Q: Does the point $x=(10, 20.25)$ satisfy the necessary optimality conditions?

A. Yes

B. No

C. I'm not sure

Lecture 25

Newton's Method

1. Unconstrained optimization

$$\text{Min } f(x)$$

where

- $x = (x_1, x_2, \dots, x_n)$
- $f(x)$ is continuous and differentiable

1. Unconstrained optimization

Necessary condition of optimality

If \bar{x} is a global minimizer of f , then

$$\nabla f(\bar{x}) = 0.$$

1. Unconstrained optimization

Example:

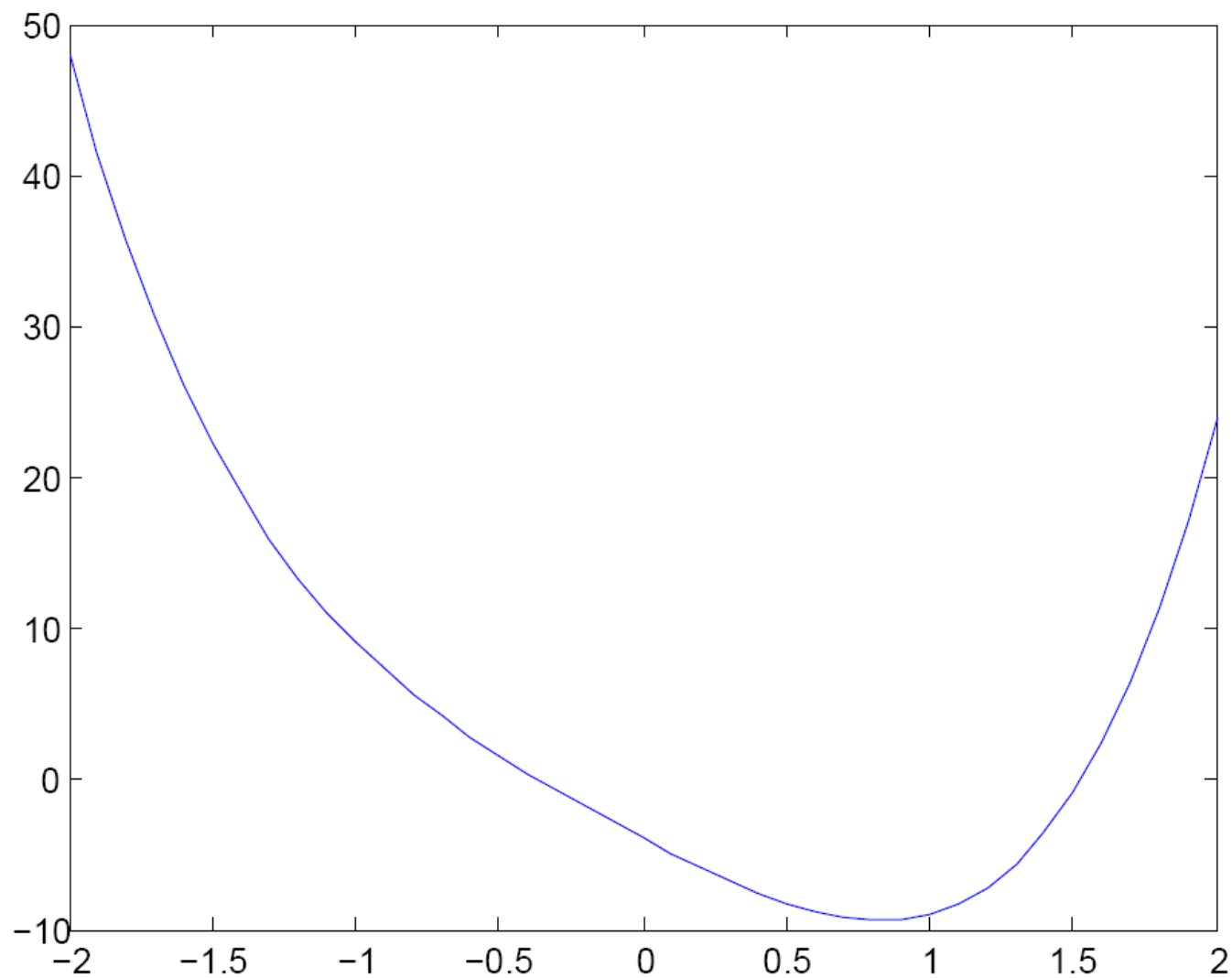
$$f(x) = 2x^2 - 10x - 5.5$$

1. Unconstrained optimization

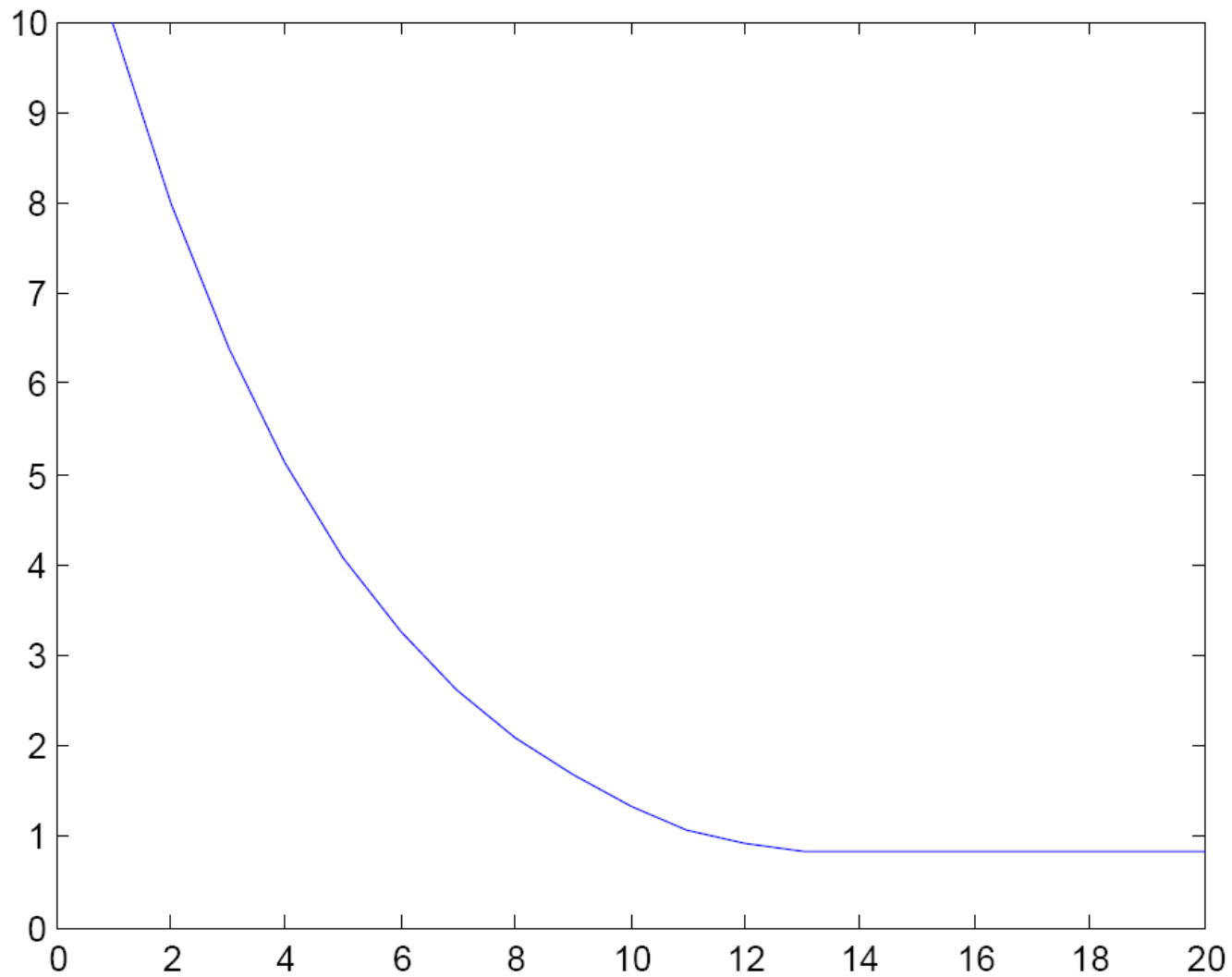
Example:

$$f(x) = 2x^6 + x^3 + 2x^2 - 10x - 5.5$$

Newton's Method



Newton's Method



1. Unconstrained optimization

Example:

$$f(x) = 2x_1^4 + 2.5x_1^2 - 2x_1x_3 + 2x_2^2 + 1.5x_3^2 \\ - 3x_1 + x_2 - 2x_3 - 2$$

1. Unconstrained optimization

