Week 12: Solving Word Problems

Step-by-step method for solving word problems

Step 1: Summarize known information from the problem statement. ("Known")

Step 2: List the unknown quantities that we must solve. ("Asked")

Step 3: Work towards solving the problem. ("Solution")

This step is the most difficult. So, here are a few more pointers:

Step 3a: What are the variables? Choose a variable for each unknown quantity (listed in "Asked")

Step 3b: What are the equations? Express the known information (listed in "Known") as equations in terms of the variables that we have just chosen

Step 3c: Solve! Using the equations, solve for the unknown quantities

Step 4: (Optional) Check if the answers make sense.

Example 1: From Page 133, Problem 44

During the first 6 months of last year, the interest on an investment was \$130 less than during the second six months. The total interest for the year was \$1450. What was the interest for each six-month period?

Step 1: Known:

- interest in first period was \$130 less than interest in second period
- total interest was \$1450

Step 2: Asked:

- interest in first period = ?
- interest in second period = ?

Step 3: Solution:

<u>What are the variables?</u>

- Let interest in first period = a.
- Let interest in second period = b.

<u>What are the equations?</u>

- "interest in first period was \$130 less than interest in second period" implies that
$$a = b - 130.$$
- " total interest was \$1450" implies that
$$a + b = 1450.$$

<u>Solve!</u>

- Since $a = b - 130$, then
$$\begin{aligned} a + b &= 1450 \\ (b - 130) + b &= 1450 \\ 2b - 130 &= 1450 \\ 2b &= 1450 + 130 \\ b &= \frac{1580}{2} = 790. \end{aligned}$$
- So,
$$a = b - 130 = 790 - 130 = 660.$$

Therefore, the interest in the first six-month period was 660 and the interest in the second six-month period was 790. (Step 4: Check: the sum is \$1450 and the interest in the first month is \$130 less than the second month.)

Example 2: From Page 133, Problem 48

Kesha drove from Buffalo to Syracuse at an average rate of 48 miles per hour. On the return trip along the same road, she was able to travel at an average rate of 60 miles per hour. The trip from Buffalo to Syracuse took one-half hour longer than the return trip. How long did the return trip take?

Step 1: Known:

- Speed from B to S was 48 mph
- Speed from S to B was 60 mph
- Time from B to S was 0.5 hour more than time from S to B.

Step 2: Asked: Time from S to B = ?

Step 3: Solution:

What are the variables?

- Let time from S to B = a.
- We are not asked for the time from B to S, but since we also don't know what it is, let time from B to S = b.

What are the equations?

- "Speed from B to S was 48 mph" implies that
$$\frac{48 \text{ miles}}{\text{hours}} = \frac{\text{distance in miles}}{b \text{ hours}}$$
- Similarly, "Speed from S to B was 60 mph" implies that
$$\frac{60 \text{ miles}}{\text{hours}} = \frac{\text{distance in miles}}{a \text{ hours}}.$$
- We now realize that we also don't know what the distance is between S and B. So, let d = distance between S and B. Therefore, our two equations from above are:
$$48 = \frac{d}{b}, \quad 60 = \frac{d}{a}.$$
- " Time from B to S was 0.5 hour more than time from S to B" implies that
$$b = a + 0.5.$$

Solve!

- Since $b = a + 0.5$, then:
$$48 = \frac{d}{a+0.5}, \quad 60 = \frac{d}{a}.$$
- The two equations are related by distance d. So:
$$\begin{aligned} 48(a+0.5) = d \quad &, \quad 60a = d \\ 48(a+0.5) &= 60a \\ 48a + 24 &= 60a \\ 24 &= 60a - 48a \\ 24 &= 12a \\ 2 &= a. \end{aligned}$$
- So,
$$b = a + 0.5 = 2.5.$$
- We can also now solve for the distance: $d = 60a = 60 \text{ mph} \times 2 \text{ hours} = 120$ miles.

Therefore, it took Kesha 2 hours to drive from Syracuse to Buffalo. Also, the trip from Buffalo to Syracuse took 2.5 hours, and the distance was 120 miles. (Step 4: Check: the trip to Buffalo was indeed 0.5 hours longer than the return trip. Also, the speed to Syracuse was 120 miles/2.5 hours = 48 mph, etc.)