Discrete Event Simulation & Queuing Systems

ORIE 3120 Lecture 10 March 3rd and 5th

Simulation

Using the computer as our laboratory!

Implement (mathematical) model of (physical) system on computer.

Do experiments on model to draw conclusions about system.

Examples

- Traffic lights
- Plant breeding
- Setting prices
- Design of cryptocurrency
- Staffing (of, e.g., call centers)
- Verifying analytic results
- https://www.youtube.com/watch?v=0ZGbIKd0XrM

There are 2 kinds of simulation

- Discrete event simulation
- Discrete time simulation
- The focus in this course will be on discrete event simulation

Overview

- Example: The Cookie Problem
- Discrete Event Simulation
- A Graphical Language
- Programming a Simulation in MS Excel

Cookie Problem (#1)

Trays of cookies arrive every 13.75 minutes



Oven cycle is 13.5 minutes, cannot be interrupted



Oven capacity is two trays

Another example



Ride sharing at an airport



Cookie Problem (#1)

Trays of cookies arrive every 13.75 minutes



Oven cycle is 13.5 minutes, cannot be interrupted



Oven capacity is two trays

- (a) 1 tray / 13.5 min = 0.074 trays/min
- (b) 2 trays / 13.5 min = 0.148 trays/min
- (c) 1 tray / 13.75 min = 0.073 trays/min
- (d) 2 trays / 13.75 min = 0.145 trays/min
- (e) none of the above

- (a) 1 tray / 13.5 min = 0.074 trays/min
- (b) 2 trays / 13.5 min = 0.148 trays/min
- (c) 1 tray / 13.75 min = 0.073 trays/min
- (d) 2 trays / 13.75 min = 0.145 trays/min
- (e) none of the above

Cookie Problem (#2)

Trays of cookies arrive every 13.75 minutes on average



Time between arrivals is uniformly distributed in [10.5,17]

A queue of trays can build up due to randomness of arrivals

Oven cycle is 13.5 minutes, cannot be interrupted



Oven capacity is two trays

- (a) 1 tray / 13.5 min = 0.074 trays/min
- (b) 2 trays / 13.5 min = 0.148 trays/min
- (c) 1 tray / 13.75 min = 0.073 trays/min
- (d) 2 trays / 13.75 min = 0.145 trays/min
- (e) none of the above

- (a) 1 tray / 13.5 min = 0.074 trays/min
- (b) 2 trays / 13.5 min = 0.148 trays/min
- (c) 1 tray / 13.75 min = 0.073 trays/min
- (d) 2 trays / 13.75 min = 0.145 trays/min
- (e) none of the above

Cookie Problem (#3)

Trays of cookies arrive every 6 minutes on average

Time between arrivals is uniformly distributed in [4,8]



A queue of trays can build up due to randomness of arrivals Oven cycle is 13.5 minutes, cannot be interrupted



Oven capacity is two trays

- (a) 1 tray / 13.5 min = 0.074 trays/min
- (b) 2 trays / 13.5 min = 0.148 trays/min
- (c) 1 tray / 6 min = 0.167 trays/min
- (d) 2 trays / 6 min = 0.333 trays/min
- (e) none of the above

- (a) 1 tray / 13.5 min = 0.074 trays/min
- (b) 2 trays / 13.5 min = 0.148 trays/min
- (c) 1 tray / 6 min = 0.167 trays/min
- (d) 2 trays / 6 min = 0.333 trays/min
- (e) none of the above

Cookie Problem (#4)

Trays of c.c. arrive every 13.5 minutes on average



uniform over [9,18]

Queues of trays can build up due to randomness of arrivals

Trays of p.b. arrive every 14 minutes on average, uniform over [12,16]



Oven cycle is 13.5 minutes but cannot be interrupted



Maximum throughput is 2 trays every 13.5 minutes



Oven capacity is two trays (can be of mixed type)

- (a) 2 trays / 13.5min = .148 trays/min
- (b) 1 tray / 13.5min + 1 tray/ 14min= .145 trays / min
- (c) 1 tray / 13.5 min = .074 trays/min
- (d) 1 tray / 14 min = .071 trays/min
- (e) none of the above

- (a) 2 trays / 13.5min = .148 trays/min
- (b) 1 tray / 13.5min + 1 tray/ 14min= .145 trays / min
- (c) 1 tray / 13.5 min = .074 trays/min
- (d) 1 tray / 14 min = .071 trays/min
- (e) none of the above



Can still answer this particular question analytically, but...

... it's easy to simulate and get an answer!

Some questions are even harder to answer analytically

- What is the average # of trays in queue?
- How often does the # of trays in queue exceed 5?

Answers may depend on the priority rule: "Always fill oven" or "cook on arrival"

• Which priority rule gives the smaller average # of trays in queue?

But again it is easy to simulate and get an answer!

Examples from ride sharing

- What fraction of riders won't be able to get a car?
- What fraction of riders will have to wait longer than 10 minutes to be picked up?
- What is the average price that riders will pay?
- How much money per hour will drivers earn?
- What pricing algorithm should I use to maximize the total value created for riders and drivers?

Again easy to simulate and get an answer!

Discrete Event Simulation

- The system is described by a state.
- The state changes only at discrete points in time, called events.
- The interval between events is called a delay, or duration. The delay could be random.
- Events can trigger other events depending on conditions that depend on the state.

The Cookie Problem

- What is the state of the system?
- At what points in time (events) does the state change?
- What are the delays?
- What events could trigger other events?
- What are the conditions under which events are triggered?

Discrete Event Simulation is powerful.

- It models complex behavior with simple language of "states", "events", "delays", "conditions", and "triggers"
- It can jump in time from one event to another
 - Nothing 'interesting' happens between events: no change in state
 - This allows it to rapidly simulate days/weeks/years of real-time activity.
- If the state is simple (e.g. inventory counts) then processing time and memory required are very small.

Discrete Event Simulation can be fast.

- Two models of semiconductor fab
 - One focused on wafer-level simulation
 - Kept track of each tray of wafers
 - One focused on machine cycles
 - Counted wafers
- Same question asked of both models
 - Is there enough capacity to meet demand?
- Same answer from both models
 - But one model (wafer counter) ran 10,000 X faster



• Example: The Cookie Problem

- A Model of Behavior: Discrete Event Simulation
- A Graphical Language
- Programming a Simulation in MS Excel

- At any point in time there is an ordered list of events scheduled to occur in the future (the event queue).
- The state of the system is described by state variables.
- The simulation engine removes the first event from the event queue and advances the simulation clock to the time recorded on that event.
- The function associated with that event is called.
 - The function may change the values of the state variables.
- The simulation engine checks to see if any trigger conditions are satisfied.
 - If a trigger condition is satisfied, the simulation engine creates a new event for each trigger.
 - The scheduled time for the new event is the current simulation clock time plus any delay associated with the trigger.
 - The new event is inserted into the event queue in order of the event time.
- The simulation continues until there are no more events in the event queue, or until a simulation stop time is reached.





Remove Next Scheduled Event










How a Discrete Event Simulation Works



How a Discrete Event Simulation Works



Remove Next Scheduled Event

Repeat

How a Discrete Event Simulation Works: Summary



The Simplified Cookie Problem

Trays of cookies arrive every 13.75 minutes on average

Time between arrivals is uniformly distributed in [10.5,17]



A queue of trays can build up due to randomness of arrivals

Oven cycle is 25 minutes but cannot be interrupted



Maximum throughput is 2 trays every 25 minutes



Oven capacity is two trays

Modeling

- State of the system
 - Q = number of trays in queue (0,1, 2,...)
 - P = number of trays in oven (0,1, or 2)
- Events that change system
 - Arrival (Q increases)
 - Start (Q decreases, P increases)
 - Finish (P decreases)
 - Initialize (set P and Q to initial values)

Modeling Triggers and Delays

- Start triggers Finish with delay of 25 minutes
 - Call this OvenCycleTime
 - OvenCycleTime = 25
- Arrival triggers Arrival with delay of 13.75 minutes, on average
 - Call this InterarrivalTime
 - InterarrivalTime =10.5 + Rnd()*(17-10.5)
 - Rnd() is a pseudo-random number in (0,1)

Modeling Conditional Triggers

- Arrival triggers Start if P=0
 - Call this condition OvenIsEmpty
 - OvenIsEmpty = if(P=0,true,false)
- Finish triggers Start if Q>0
 - Call this condition CookiesInQueue
 - CookiesInQueue = if (Q>0,true,false)

Modeling State Changes

- **Arrival**: Q = Q+1
- **Finish**: P = 0
- **Start**: if Q > 2 then P = 2, else P = Q; Q = Q-P
 - Start does not get triggered unless Q is at least 1
- Initialize: Q=0,P=0



Motivation: The Cookie Problem



A Model of Behavior: Discrete Event Simulation

- Event Graph Language
- Programming a Simulation in MS Excel

Event Graph Language

We will use Excel drawing tools to describe discrete event simulations using the event graph language



Graph Language is for Illustration Purposes

Graph language implements event, trigger and delay logic of simulation

Could also implement this yourself in any general purpose language (e.g., Python)

[Still have to implement state changes in VBA]

Next few slides uses Excel Implementation

... but the main purpose is to serve as an example what Discrete Event Simulation is.

The Simplified Cookie Model



The Simplified Cookie Model



First Event



Second Event



What does the event queue look like right now?

Second Event



- Start at t=0, Finish at t=25
 - Start at t=0
- Arrival at a time randomly distributed between 10.5 and 17
- Start at t=0, Arrival at a time randomly distributed between 10.5 and 17 Finish at t=25

What does the event queue look like right now?

Second Event



- (a) Start at t=0, Finish at t=25
 - Start at t=0
 - Arrival at a time randomly distributed between 10.5 and 17
 - Start at t=0, Arrival at a time randomly distributed between 10.5 and 17
 - Finish at t=25

Third Event



What does the event queue look like right now?

Third Event



- (a) Arrival at a time randomly distributed between 10.5 and 17; Finish at t=25
- (b) Arrival at a time randomly distributed between 10.5 and 17; Finish at t=0
- (c) Arrival at a time randomly distributed between 10.5 and 17; Finish at a time randomly distributed between 0 and 25
- (d) Finish at a time randomly distributed between 0 and 25
- (e) Finish at t=25

What does the event queue look like right now?

Third Event



- (a) Arrival at a time randomly distributed between 10.5 and 17; Finish at t=25
- (b) Arrival at a time randomly distributed between 10.5 and 17; Finish at t=0
- (c) Arrival at a time randomly distributed between 10.5 and 17; Finish at a time randomly distributed between 0 and 25
- (d) Finish at a time randomly distributed between 0 and 25
- (e) Finish at t=25

Fourth Event



What does the event queue look like right now?

Fourth Event



- (a) Arrival at time t+Uniform(10.5,17);Finish at time 25
 -) Finish at time 25
 -) Arrival at time t+Uniform(10.5,17)
 - Arrival at time t+Uniform(10.5,17), Start at time t
 -) Arrival at time t+Uniform(10.5,17); Finish at time 25, Start at time t

What does the event queue look like right now?

Fourth Event



- (a) Arrival at time t+Uniform(10.5,17); Finish at time 25
 - Finish at time 25
 - Arrival at time t+Uniform(10.5,17)
 - Arrival at time t+Uniform(10.5,17), Start at time t
- (e) Arrival at time t+Uniform(10.5,17); Finish at time 25, Start at time t

Fifth Event



Graph Rules

- There must be one event node that has no incoming arcs (triggers): this is the first event
- Nodes must be connected (use MS Excel connectors).
- Condition nodes trigger all outgoing arcs, if condition is true
 - There are no "yes/no" branches in this language
 - You will need <u>two</u> condition nodes to model a branching process (one for the "yes" and one for the "no")
- Events can be triggered only by other events, through condition nodes and delay nodes

Yes-No Branches are not allowed Instead use two conditions



In Recitation & HW, you will get hands-on experience

- First, you will create an MS Excel style event graph to model a problem
- Then, you will program and run the simulation model.



Motivation: The Cookie Problem





Programming a Simulation in MS Excel

Now we'll focus on this particular Excel implementation

- Use Visual Basic for Applications (VBA) in Excel
- Use simulation template file to start
 - "SimplifiedCookie.xls"
 - Contains code to analyze graph and run simulation
 - Simplified model is already coded

Building the Graphical Model

Model in Excel



Use Tokens in Text Strings

- Template code ignores shape
- Use tokens to indicate type of node
 - = for events
 - + for delays
 - ? for conditions

Model With Tokens



Analyze and Run Dialog



English Interpretation on Sheet "SimLog"

1	A
1	Model structure is valid. An English interpretation of your model follows:
2	
3	When event Initialize() occurs, after this, event Arrival happens.
4	
	When event Arrival() occurs, after this, a delay of duration InterarrivalTime() takes place; after this, event Arrival
5	happens. Also, if OvenIsEmpty() then after this, event Start happens.
6	
	When event Start() occurs, after this, a delay of duration OvenCycleTime() takes place; after this, event Finish
7	happens.
8	
9	When event Finish() occurs, after this, if CookiesInQueue() then after this, event Start happens.
10	
11	
ŔĨ	Image: Sheet1 / Sheet2 / Sheet3 / SimLog / SimTrace / Image: Sheet3 / SimLog / SimTrace /
Coding the Components

Switch to Visual Basic Editor (<Alt><F11>)



Development Code: Do not modify (all variables and objects beginning with "Sim" are reserved)

 User Code: Put your code here. Add more modules if you like

You Write the Code: Declare Your State Variables

'Declare your variables here Global Q As Integer Global P As Integer Global CumulativeCompletions As Integer

Global means it is available for use in other modules.

Q is the variable tracking the number of trays in the queue.

P is the number of trays in the oven.

CumulativeCompletions is a statistic we want to compute; it is not essential to the model.

Write the Event Functions (to change the state)

Function Initialize() 'every simulation should have a function which initializes the state variables Q = 0 P = 0 CumulativeCompletions = 0	
End Function Function Arrival() 'this represents the arrival of a tray of cookies Q = Q + 1 End Function	Function Start() 'this function represents the start of the oven cycle If $Q > 2$ Then $P = 2$ Else $P = Q$ Q = Q - P End Function
	Function Finish() 'this function represents the end of the oven cycle CumulativeCompletions = CumulativeCompletions + P P = 0 End Function

Write the Condition Functions: to Test the State

Function CookiesInQueue() As Integer

'this illustrates the if...then...else statement If Q > 0 Then CookiesInQueue = True Else CookiesInQueue = False End Function

Function OvenIsEmpty() As Integer

If P = 0 Then OvenIsEmpty = True Else OvenIsEmpty = False End Function

> Condition functions must return an integer: True = -1; False = 0

Question

Which of these lines of code returns True if N < 5 inside of a function called F?

- (a) If N<5 Then Return True Else Return False
- (b) If N<5 Then F=True Else F=False
- (c) If N<5 Return True Else Return False
- (d) If N<5 F=True Else F=False
- (e) None of the above



Which of these lines of code returns True if N < 5 inside of a function called F?

(a) If N<5 Then Return True Else Return False

(b) If N<5 Then F=True Else F=False

- (c) If N<5 Return True Else Return False
- (d) If N<5 F=True Else F=False
- (e) None of the above

Question

Which of these lines of code returns True if N < 5 inside of a function called F?

- (a) If N<5 Then F=-1 Else F=0
- (b) If N<5 Then F=1 Else F=0
- (c) If N<5 Then F=1 Else F=-1
- (d) If N<5 Then F=0 Else F=-1
- (e) None of the above

Question

Which of these lines of code returns True if N < 5 inside of a function called F?

(a) If N<5 Then F=-1 Else F=0

- (b) If N<5 Then F=1 Else F=0
- (c) If N<5 Then F=1 Else F=-1
- (d) If N<5 Then F=0 Else F=-1
- (e) None of the above

Write the Code to Generate Delays and Durations

Function OvenCycleTime() As Variant

'functions that return a value for time should use the Variant data type OvenCycleTime = 25 End Function

Function InterarrivalTime() As Variant

'this function returns a random interarrival time Dim duration As Variant 'local variable declaration; duration will be the length of the interarrival time duration = 10.5 + Rnd() * 6.5 'duration will be a random number uniformly distributed between 10.5 and 17.

InterarrivalTime = duration 'this is how you return a value

End Function

Question

How do I create a random variable that is Uniformly distributed between 5 and 7?

- (a) duration = 5 + 7*Rnd()
- (b) duration = 7 + 5*Rnd()
- (c) duration = 5 + 2*Rnd()
- (d) duration = 2 + 5*Rnd()
- (e) None of the above

Debugging the Model

Debug: Step Through Model



Define Ranges to Store Variables



Write Code to Store Variables

Function OutputVariables()	
Worksheets("Sheet1").Range("Number_of_Trays	s_in_Queue").Value = Q
Worksheets("Sheet1").Range("Number_of_Trays	s_in_Oven"). Value = P
Worksheets("Sheet1").Range("Cumulative_Comp	pletions").Value = CumulativeCompletions
End Function	
Your range names	Your state variables

Modify Code to Store Variables After Each Event

```
Function Initialize()

'every simulation should have a function which initializes the state variables

Q = 0

P = 0

Cumulative Completions = 0

Output Variables

End Function
```

```
Function Arrival()
'this represents the arrival of a tray of cookies
Q = Q + 1
OutputVariables
End Function
```

```
Function Start()

'this function represents the start of the oven cycle

If Q > 2 Then P = 2 Else P = Q

Q = Q - P

OutputVariables

End Function
```

```
Function Finish()

'this function represents the end of the oven cycle

CumulativeCompletions = CumulativeCompletions + P

P = 0

OutputVariables

End Function
```

Input/Output

- If you want to read input data from the spreadsheet (eg. Initial parameter settings), use ranges in a similar way.
- Now, single step through your simulation watching your variables change with each event.
 - The more variables you track, the easier it will be to debug your model.

First Event



Second Event



Third Event



Fourth Event



Fifth Event



Running the Model, Collecting its Output

Creating a Trace

- A trace is a history of your state variables after each event
- The simulator automatically writes out whatever is in the range called "SimTraceRange" before and after each event
 - Stored on separate lines of sheet "SimTrace"
- It also writes out the labels found in the range called "SimTraceLabelRange" at the head of this list.
- You must define these two ranges.

Define Trace and Label Ranges



This is how a trace looks.

	A	В	C	D	E	F	0	
1	Event Name	Elapsed Time	Current Time	Number of Trays in Queue	Number of Trays in Oven	Cumulative Completions		
2			Max	3	2	365		
3			Min	0	0	0		
4			Mean	0.913120476	1.824999926	181.0799999		
5			Std. Dev.	0.6813104	0.379967273	106.1439287		
6		0.000001	1E-06					
7	Initialize	0	1E-06	0	0	0		
8	Initialize	0.000001	2E-06	0	0	0		
9	Arrival	0	2E-06	1	0	0		
10	Arrival	0.000001	3E-06	1	0	0		
11	Start	0	3E-06	0	1	0		Thor
12	Start	15.086058	15.0861	0	1	0		📕 i nere
13	Arrival	0	15.0861	1	1	0		
14	Arrival	9.9139422	25	1	1	0		each
15	Finish	0	25	1	0	1		
16	Finish	1E-06	25	1	0	1		end
17	Start	0	25	0	1	1		01101.
18	Start	4.053313	29.0533	0	1	1		canti
19	Arrival	0	29.0533	1	1	1		Capit
20	Arrival	14.266871	43.3202	1	1	1		ctata
21	Arrival	0	43.3202	2	1	1		Sidle
22	Arrival	6.679816	50	2	1	1		
23	Finish	0	50	2	0	2		

There are 2 rows for each event: begin and end. The 2nd row captures time spent in state ("Elapsed time")

This is how a trace looks.

	A B		C	D	E	F	(
1	Event Name	Elapsed Time	Current Time	Number of Trays in Queue	Number of Trays in Oven	Cumulative Completions	
2			Max	3	2	365	
3			Min	0	0	0	
4			Mean	0.913120476	1.824999926	181.0799999	
5			Std. Dev.	0.6813104	0.379967273	106.1439287	
6		0.00000	1E-06				
7	Initialize		1E-06	0	0	0	
8	Initialize	0.00000	2E-06	0	0	0	
9	Arrival		2E-06	1	0	0	
10	Arrival	0.00000	3E-06	1	0	0	
11	Start		3E-06	0	1	0	
12	Start	15.08605	15.0861	0	1	0	
13	Arrival		15.0861	1	1	0	
14	Arrival	9.913942	25	1	1	0	
15	Finish		25	1	0	1	
16	Finish	1E-0	25	1	0	1	
17	Start		25	0	1	1	
18	Start	4.05331	29.0533	0	1	1	
19	Arrival		29.0533	1	1	1	
20	Arrival	14.26687	43.3202	1	1	1	
21	Arrival		43.3202	2	1	1	
22	Arrival	6.67981	50	2	1	1	
23	Finish		50	2	0	2	

To see how a variable changes over time, make an X-Y scatter plot. Get X from "Current Time" and Y from the variable you want to plot (e.g., "Number of Trays in Queue")

Analyzing the Output

X-Y Scatter Plot



Statistics Computed After Each Run

- Statistics inserted into first four lines of trace output.
- Four statistics computed (Min, Max, Mean, Std. Dev.) even if they don't make sense for your particular state variables

96 	A	В	C	D	E	F
1	Event Name	Elapsed Time	Current Time	Number of Trays in Queue	Number of Trays in Oven	Cumulative Completions
2			Max	3	2	365
3			Min	0	0	0
4			Mean	0.913120476	1.824999926	181.0799999
5			Std. Dev.	0.6813104	0.379967273	106,1439287

One Problem

- If you copy and paste a node or a connector, MS Excel does not give it a new name.
- Since the code uses the name of the autoshape to identify it, the code gets confused.
- Be sure to create each node from the shapes menu fresh, to make sure it has a unique name.



Motivation: The Cookie Problem





Programming a Simulation in MS Excel

Key Lessons

- Discrete event simulation is a flexible way to describe a system's behavior.
- Basic simulations can be implemented in Excel, but require user coding in VBA.
- The structure of a simulation model can be described with an event graph.
- Building, running, and analyzing a simulation model is a structured process.