# ORIE 3120: Industrial Data and Systems Analysis <br> Spring 2020 <br> Homework \#4 <br> Due Date: 2/28/2020 (Friday) 2:30pm 

Upload 1 file to gradescope: A pdf containing the answers to all questions and screenshots of the .xls or .xlsx file containing the spreadsheet created in question 4.

Question 1 ( 25 points). The Ithaca Science Center is ordering t-shirts to sell in their souvenir shop. Each t-shirt costs the Science Center $\$ 7.00$ to purchase. The holding cost per $t$-shirt is due solely to lost interest, and is $\$ 2.00$ per year. Each order costs $\$ 50$ to process and ship. Demand for the $t$-shirts is constant over time, and is 350 per year. Each $t$-shirt sells for $\$ 12.99$ each. The lead time is 3 weeks.
a) Find the optimal number of t-shirts that the Science Center should order in each batch.
b) Find the inventory level that would trigger a replenishment order.
c) Find the average inventory level under the optimal order quantity.
d) What is the smallest price that the Science Center can charge for their t-shirts and not lose money? Assume that they operate their system using the optimal order quantity from (a).
e) Suppose that the supplier requires $t$-shirts to be ordered in multiples of 10 shirts. What is the optimal order quantity that meets this requirement?

## Question 2 (25 points)

In the analysis of EOQ in class, we assumed that the cost of placing an order for x units was $\mathrm{c}(\mathrm{x})$ $=K+c x$. What is the optimal order quantity if the cost is $c(x)=K+d^{2}$ ?

## Question 3 (25 points)

In the analysis of EOQ in class, we assumed that the cost per unit time for holding $x$ units was hx . What is the optimal order quantity if the cost per unit time for holding x units is $\mathrm{hx}^{2}$ ?

## Question 4 (25 points)

Create an Excel spreadsheet that implements the algorithm on slide 37 Lecture 8 (the bisection algorithm). Given this mean $m$, and a number $q$ strictly between 0 and 1, this spreadsheet should find an integer $Q^{*}$ such that $P\left(D<=Q^{*}\right)>=q$ and $P\left(D<=Q^{*}-1\right)<q$.

Assume that $D$ has the Poisson distribution. The Poisson distribution is a discrete probability distribution often used for modeling demand. The probability mass function for a Poisson with mean $m$ is $P(D=k)=m^{k} \exp (-m) / m!$, for $k=0,1,2, \ldots$ The cumulative distribution function for D can be computed using Excel's POISSON.DIST function. Take a moment to read about

POISSON.DIST in Excel's online help to see how it works. For example, to compute $P(D<=13)$ when D has a mean of 10, use POISSON.DIST(13,10,TRUE).

Here is a screenshot of what your completed spreadsheet will look like, run using $\mathrm{q}=0.5$ and a Poisson distribution with mean 10.

| - |  | A | B | C | D | E |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 |  | 10 | <-Mean of Poisson |  |  |  |
| 2 |  | 0.5 | <-q |  |  |  |
| 3 |  |  |  |  |  |  |
| 4 | L |  | U | Q | $\mathrm{P}(\mathrm{D}<=\mathrm{Q})$ | Done? |
| 5 |  | 0 | 100 | 50 | 1 | No |
| 6 |  | 0 | 50 | 25 | 0.99998232 | No |
| 7 |  | 0 | 25 | 13 | 0.86446442 | No |
| 8 |  | 0 | 13 | 7 | 0.22022065 | No |
| 9 |  | 7 | 13 | 10 | 0.58303975 | No |
| 10 |  | 7 | 10 | 9 | 0.45792971 | No |
| 11 |  | 9 | 10 | 10 | 0.58303975 | Yes |
| 12 |  | 9 | 10 | 10 | 0.58303975 | Yes |
| 13 |  | 9 | 10 | 10 | 0.58303975 | Yes |
| 14 |  | 9 | 10 | 10 | 0.58303975 | Yes |
| 15 |  | 9 | 10 | 10 | 0.58303975 | Yes |
| 16 |  | 9 | 10 | 10 | 0.58303975 | Yes |
| 17 |  | 9 | 10 | 10 | 0.58303975 | Yes |
| 18 |  | 9 | 10 | 10 | 0.58303975 | Yes |
| 19 |  | 9 | 10 | 10 | 0.58303975 | Yes |

This solution uses these Excel functions, which you can read about in the Excel help or online:

- POISSON.DIST
- ROUND
- IF

It starts with initial values for $L$ and $U$ in row 4 (the value of $U$ must be chosen large enough that $P(D<=Q)$ is above $q)$, and with the mean of the Poisson and $q$ in rows 1 and 2. In each row, columns $C, D$, and $E$ are computed from the values to their left, and columns $A$ and $B$ are computed from values above. When the spreadsheet has "Yes" in the Done column, the solution is found in the U column.

Use your spreadsheet to find $Q^{*}$ for a Poisson distribution with mean 10 for $q=0.7$. include a screenshot of your spreadsheet along with your calculated value for $Q^{*}$ in your pdf submission.

