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Recitation 6:

Finding initial solution:

Example 1: min —

$$\text{s.t.} \quad 2x_1 - x_2 \geq 4$$
$$x_1 + x_3 = 2$$

introduce artificial variables x_5 and x_6 and surplus var. x_4

$$\min \quad x_5 + x_6$$
$$\text{s.t.} \quad 2x_1 - x_2 - x_4 + x_5 = 4$$
$$x_1 + x_3 + x_6 = 2$$
$$x \geq 0$$

z	x_1	x_2	x_3	x_4	x_5	x_6	RHS
1	0	0	0	0	-1	-1	0
x_5	2	-1	0	-1	1	0	4
x_6	1	0	1	0	0	1	2

we want obj funct coef. = 0 for basic variables!

so our initial tableau is:

	z	x_1	x_2	x_3	x_4	x_5	x_6	RHS
	1	3	-1	1	-1	0	0	6
x_5	0	2	-1	0	-1	1	0	4
x_6	0	1	0	1	0	0	1	2
	1	0	-1	-2	-1	0	-3	0
x_5	0	0	-1	-2	-1	1	-2	0
x_1	0	1	0	1	0	0	1	2

feasible solution but not basic feasible for original since x_5 is still basic (=0)

We can pivot and take x_5 out basis.

z	x_1	x_2	x_3	x_4	x_5	x_6	RHS
1	0	0	0	0	-1	-1	0
x_2 0	0	1	2	1	-1	2	0
x_1 0	1	0	1	0	0	1	2

$\longrightarrow X = \begin{pmatrix} 2 \\ 0 \\ 0 \\ 0 \end{pmatrix}$ is the initial basic feasible solution.

basis $= \{x_1, x_2\}$, $B^{-1} = \begin{bmatrix} -1 & 2 \\ 0 & 1 \end{bmatrix}$

Now continue with original problem, using

<u>Example 2:</u>

$$\begin{aligned} \min \quad & \\ \text{s.t.} \quad & 2x_1 - x_2 \geq 4 \\ & x_1 + x_3 = 2 \\ & 2x_1 + 2x_3 = 4 \end{aligned}$$

$\longrightarrow$ (note this constraint is redundant $\operatorname{rank}(A) = 2 < m = 3$, but it is <u>not</u> always easily to find it by inspection!)

introduce surplus var x_4 and artifical variables x_5, x_6 and x_7.

$$\begin{aligned} \min \quad & x_5 + x_6 + x_7 \\ \text{s.t.} \quad & 2x_1 - x_2 - x_4 + x_5 = 4 \\ & x_1 + x_3 + x_6 = 2 \\ & 2x_1 + 2x_3 + x_7 = 4. \end{aligned}$$

	z	x_1	x_2	x_3	x_4	x_5	x_6	x_7	RHS
	1	0	0	0	0	-1	-1	-1	0
x_5	0	2	-1	0	-1	1	0	0	4
x_6	0	1	0	1	0	0	1	0	2
x_7	0	2	0	2	0	0	0	1	4

	z	x_1	x_2	x_3	x_4	x_5	x_6	x_7	RHS
	1	5	-1	3	-1	0	0	0	10
x_5	0	2	-1	0	-1	1	0	0	4
x_6	0	1	0	1	0	0	1	0	2
x_7	0	2	0	2	0	0	0	1	4
	1	0	-1	-2	-1	0	-5	0	0
x_5	0	0	-1	-2	-1	1	-2	0	0
x_1	0	1	0	1	0	0	1	0	2
~~x_7~~	~~0~~	~~0~~	~~0~~	~~0~~	~~0~~	~~0~~	~~-2~~	~~1~~	~~0~~

→ This is a row involving only the artificial variables: $x_7 = 2x_6$ for all feasible solutions, means the last equation is exactly twice the one before. So it shows the rank of the matrix A is 2, not 3, and you can eliminate the dependent row, hence delete the last row & corr. basic var.