Recitation 6:

Finding initial solution:

Example 1: min \[ 2x_1 - x_2 \geq 4 \]

subject to \[ x_1 + x_3 = 2 \]

introduce artificial variables \( x_5 \) and \( x_6 \) and surplus var. \( x_4 \)

min \[ x_5 + x_6 \]

subject to \[ 2x_1 - x_2 - x_4 + x_5 = 4 \]

\[ x_1 + x_3 + x_6 = 2 \]

\[ x \geq 0 \]

so our initial tableau is:

\[
\begin{array}{cccccc|c}
2 & x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & \text{RHS} \\
1 & 0 & 0 & 0 & 0 & -1 & -1 & 0 \\
1 & 0 & 1 & 0 & 0 & 1 & 0 & 4 \\
1 & 0 & 0 & 0 & 0 & 1 & 0 & 2 \\
\end{array}
\]

we want obj funct coefs. = 0 for basic variables!

\[
\begin{array}{cccccc|c}
= & x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & \text{RHS} \\
1 & 0 & 1 & -1 & -1 & 0 & 0 & 6 \\
1 & 0 & 0 & 1 & 0 & 0 & 1 & 2 \\
\end{array}
\]

feasible solution but not basic feasible for original since, \( x_5 \) is still basic (≠0)
We can pivot and take $x_5$ out of the basis:

<table>
<thead>
<tr>
<th></th>
<th>$X_1$</th>
<th>$X_2$</th>
<th>$X_3$</th>
<th>$X_4$</th>
<th>$X_5$</th>
<th>$X_6$</th>
<th>RHS</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>-1</td>
<td>-1</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>$X_5$</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>$X_6$</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>

$X = \begin{pmatrix} 2 \\ 0 \\ 0 \end{pmatrix}$ is the initial basic feasible solution $x' = \begin{pmatrix} -1 & 1 & 2 \end{pmatrix}$

Now continue with the original problem, using basis $\{x_1, x_2\}$, $x' = \begin{pmatrix} 0 & 1 \end{pmatrix}$

**Example 2:**

\[
\begin{align*}
\min & \quad 2x_1 - x_2 \\
\text{s.t.} & \quad x_1 + x_3 = 2 \\
& \quad 2x_1 + 2x_3 = 4
\end{align*}
\]

(Note this constraint is redundant, $\text{rank}(A) = 2 < m = 3$, but it is not always easily to find it by inspection.)

Introduce surplus var $x_4$ and artificial variables $x_5, x_6$ and $x_7$.

\[
\begin{align*}
\min & \quad x_5 + x_6 + x_7 \\
\text{s.t.} & \quad 2x_1 - x_2 - x_4 + x_5 = 4 \\
& \quad x_1 + x_3 + x_6 = 2 \\
& \quad 2x_1 + 2x_3 + x_7 = 4
\end{align*}
\]

<table>
<thead>
<tr>
<th></th>
<th>$X_1$</th>
<th>$X_2$</th>
<th>$X_3$</th>
<th>$X_4$</th>
<th>$X_5$</th>
<th>$X_6$</th>
<th>$X_7$</th>
<th>RHS</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>-1</td>
<td>-1</td>
<td>-1</td>
<td>0</td>
</tr>
<tr>
<td>$x_5$</td>
<td>0</td>
<td>2</td>
<td>-1</td>
<td>0</td>
<td>-1</td>
<td>1</td>
<td>0</td>
<td>4</td>
</tr>
<tr>
<td>$x_6$</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>$x_7$</td>
<td>0</td>
<td>2</td>
<td>0</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>Z</td>
<td>x₁</td>
<td>x₂</td>
<td>x₃</td>
<td>x₄</td>
<td>x₅</td>
<td>x₆</td>
<td>x₇</td>
<td>RHS</td>
</tr>
<tr>
<td>----</td>
<td>----</td>
<td>----</td>
<td>----</td>
<td>----</td>
<td>----</td>
<td>----</td>
<td>----</td>
<td>-----</td>
</tr>
<tr>
<td>1</td>
<td>5</td>
<td>-1</td>
<td>3</td>
<td>-1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>10</td>
</tr>
<tr>
<td>x₅</td>
<td>0</td>
<td>2</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>x₆</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>x₇</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>

This is a row involving only the artificial variables:

\[ x₇ = 2x₆ \]

For all feasible solutions, means the last equation is exactly twice the one before. So it shows the rank of the matrix A is 2, not 3, and you can eliminate the dependent row, hence delete the last row & corr. basic var.