Stochastic Control of Path Optimization for Inter-Switch Handoffs in Wireless ATM Networks

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Abstract—One of the major design issues in wireless ATM networks is the support of inter-switch handoffs. An inter-switch handoff occurs when a mobile terminal moves to a new base station connected to a different switch. Apart from resource allocation at the new base station, an inter-switch handoff also requires connection rerouting. With the aim of minimizing the handoff delay while using the network resources efficiently, the two-phase handoff protocol uses path extension for each inter-switch handoff, followed by path optimization if necessary. The objective of this paper is to determine when and how often path optimization should be performed. The problem is formulated as a semi-Markov decision process. Link cost and signaling cost functions are introduced to capture the trade-off between the network resources utilized by a connection and the signaling and processing load incurred on the network. The time between inter-switch handoffs follows a general distribution. A stationary optimal policy is obtained when the call termination time is exponentially distributed. Numerical results show significant improvement over four other heuristics.

Keywords—connection rerouting, path optimization, inter-switch handoff, wireless ATM. 1

I. INTRODUCTION

In recent years, there has been active research in supporting terminal mobility in multimedia broadband networks. One of the design issues is the support of inter-switch handoffs. Handoffs occur when a mobile terminal moves from one base station to another. Handoffs for multimedia traffic differ from conventional voice handoffs in that a mobile user may have several active connections with different bandwidth requirements and quality-of-service (QoS) constraints. The handoff function should ensure that all ongoing connections are rerouted to another access point in a seamless manner. In other words, the design goal is to prevent service disruption and degradation during and after the handoff process.

Handoff in general can be divided into two classes, namely intra-switch handoffs and inter-switch handoffs.

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necessary. During path optimization, the network determines the optimal path between the source and the destination (i.e., the path between the remote terminal and the current target switch in Figure 2) and transfers the user information from the old path to the new path. The major steps in the path optimization process generally involve [10]: (1) determining the location of the crossover switch; (2) setting up a new branch connection; (3) transferring the user information from the old branch connection to the new one; and (4) terminating the old branch connection.

Since the mobile terminal is still communicating over the extended path via the current base station while path optimization takes place, this gives enough time for the network to perform the necessary functions while minimizing any service disruptions. Notice that the path optimization process described above is not restricted to the two-phase handoff protocol. It can also be applied to other connection rerouting protocols where the end-to-end path after rerouting is sub-optimal. In addition, when the mobile terminal moves to another switch during the execution of path optimization, path extension can still be used to extend the connection to the target switch. To ensure a seamless path optimization, three important issues need to be addressed: 1. How to determine the location of the crossover switch? 2. How can the service disruptions be minimized during path optimization? 3. When and how often should path optimization be performed?

For the first issue, a crossover switch determination algorithm based on PNNI (Private Network-to-Network Interface) standard is proposed in [10]. Five different crossover switch algorithms for wireless ATM local area networks are proposed in [4]. For the second issue, cell loss and cell missequencing can be prevented by using appropriate signaling and buffering at the anchor and crossover switches during path optimization [10].

In this paper, we focus on the third issue. Our work is motivated by the fact that path optimization does not have to be performed after each inter-switch handoff. Although path optimization can increase the network utilization by rerouting the connection to a more efficient route, transient QoS degradations such as cell loss and an increase in cell delay variation may occur. In addition, if there are a large number of mobile users with high movement patterns, performing path optimization after each path extension will increase the processing load of certain switches and the signaling load of the network. The decision to perform path optimization should be based on several factors including: the amount of network resources (e.g., bandwidth) utilized by the connection; QoS requirements; the remaining time of the connection; and the signaling load of the network.

To this end, we propose a stochastic model to determine the optimal time to perform path optimization for the two-phase handoff protocol. The path optimization problem is formulated as a semi-Markov decision process [13]. Link cost and signaling cost functions are introduced to capture the trade-off between the network resources utilized by a connection and the signaling and processing load incurred on the network. The objective is to determine the optimal policy which minimizes the expected total cost per call. The major contribution of our work lies in the formulation of a general model that is applicable to a wide range of conditions. Distinct features of our model include: (1) different link cost functions can be assigned to different service classes (e.g., CBR, VBR, ABR) with different bandwidth requirements; (2) different signaling cost functions can be used based on the complexity of the path optimization procedures and the signaling load of the network; and (3) the time between inter-switch handoffs can follow an arbitrary general distribution.

The rest of the paper is organized as follows. The model formulation of the path optimization problem is described in Section II. In Section III, we describe the optimality equations, the value iteration algorithm, and the structure of the optimal policy. The implementation issues are described in Section IV. Extension of the model to include mobile-to-mobile connections and other QoS constraints are described in Section V. In Section VI, we present numerical results and compare the optimal policy with four other heuristics. Conclusions are given in Section VII.

II. MODEL FORMULATION

Each mobile connection may experience a number of inter-switch handoffs during its connection lifetime. During each inter-switch handoff, path extension can be used to extend the connection from the current anchor switch to the target switch. Although path extension is simple to implement, the connection utilizes more network resources than necessary. Occasional path optimization is required to reroute the connection to an optimal path. Path optimization is a complex process. It increases the processing and signaling load of the network. Thus, there is a trade-off between the network resources utilized by the connection and the processing and signaling load incurred on the network. We formulate the above problem as a semi-Markov decision process. After each path extension, the network must decide whether to perform subsequent path optimization. The decision is based on the current number of links of the path and the locations of the anchor and switch.

The model is described below.

A. Semi-Markov Decision Process Model

When an inter-switch handoff occurs, a path extension is performed. After that, a decision must be made whether to perform subsequent path optimization. These time instants are called decision epochs. Referring to Figure 3, the sequence $\sigma_0, \sigma_1, \ldots$ represents the time of successive decision epochs. Let $\tau_n = \sigma_n - \sigma_{n-1}$ for $n \geq 1$. Since inter-switch handoff only occurs during the call lifetime, the time interval requiring mobility monitoring is between the call arrival and its termination. The term $\sigma_0 = 0$ represents the arrival time of a new call and the random variable $T$ denotes the call termination time. The random variable $\phi(T)$ denotes the total number of inter-switch handoffs that occur before call termination time $T$.

At each decision epoch, the network must decide whether
to perform subsequent path optimization. Let $A = \{NPO, PO\}$ denote the action set, where PO corresponds to “perform path optimization after path extension,” and NPO corresponds to “perform path extension only.” We use $Y_n$ to denote the action chosen at decision epoch $n$.

The action chosen is based on the current state of the connection. The state space is denoted by $S$. For each state $s \in S$, the state information includes the locations of the target and anchor switches, and the number of links of the current path. The random variable $X_n$ denotes the state at decision epoch $n$.

Two cost functions are introduced to account for the network resources utilized and the signaling load incurred due to an inter-switch handoff. The link cost function reflects the amount of network resources used during the connection lifetime, while the signaling cost function captures the processing and signaling load incurred on the network due to path extension and path optimization. The signaling costs are incurred only at the decision epochs, while the link cost is accrued continuously over the call lifetime.

The function $f(s)$ denotes the link cost rate in state $s$. If the state is equal to $s$ during the time interval $(\sigma_n, \sigma_{n+1})$, then the link cost incurred during that period is equal to $(\sigma_{n+1} - \sigma_n) f(s)$. The function $b(s, a)$ denotes the signaling cost incurred when the decision maker chooses action $a$ in state $s$. Thus, $b(s, NPO)$ represents the signaling cost of performing path extension, and $b(s, PO)$ represents the signaling cost of performing path extension and subsequent path optimization. All cost functions are assumed to be finite and nondecreasing with respect to the number of links of the current path.

A decision rule prescribes a procedure for action selection in each state at a specified decision epoch. Deterministic Markovian decision rules are functions $\delta_t : S \rightarrow A_s$ that specify the action choice when the system occupies state $s$ at decision epoch $t < T$. That is, for each $s \in S$, $\delta_t(s) \in A_s$. This decision rule is said to be Markovian because it depends on previous system states and actions only through the current state of the system, and deterministic because it chooses an action with certainty. A policy $\pi$ specifies the decision rules to be used at all decision epochs. That is, a policy is a sequence of decision rules, $\pi = (\delta_1, \delta_2, \ldots)$. The set of all policies is denoted by $\Pi$.

Let $v^\pi(s)$ denote the expected total cost per call given policy $\pi$ is used with initial state $s$. Thus,

$$v^\pi(s) = E^\pi\left\{ \sum_{n=0}^{\phi(T)-1} b(X_n, Y_n) + \sum_{n=0}^{\phi(T)-1} \left[ (\sigma_{n+1} - \sigma_n) f(X_n) \right] + (T - \sigma_{\phi(T)}) f(X_{\phi(T)}) \right\}$$

where $E^\pi$ denotes the expectation with respect to policy $\pi$ and initial state $s$. In (1), the first summation in the right hand side corresponds to the lump sum portion of the signaling cost, each term in the second summation corresponds to the continuous portion of the link cost incurred at rate $f(X_n)$ between decision epochs $n$ and $n+1$, and the last term corresponds to the link cost incurred at rate $f(X_{\phi(T)})$ between decision epoch $\phi(T)$ and termination time $T$. In this paper, we assume the cost, transition probabilities, and sojourn times are time homogeneous and that the call termination time is exponentially distributed with rate $\mu$. In that case, (1) can be written as:

$$v^\pi(s) = E^\pi \left\{ \sum_{n=0}^{\phi(T)-1} e^{-\mu\sigma_n} c(X_n, Y_n) \right\}$$

where

$$c(s, a) = b(s, a) + E^\pi \left\{ \frac{1}{\mu} (1 - e^{-\mu\tau}) f(s) \right\}.$$  

(3)

Due to space limitation, please refer to [14] for a proof of this fact. The expression in (2) is the expected total cost of an infinite-horizon semi-Markov decision process with discount rate $\mu$. The function $c(s, a)$ in (3) is the expected total cost between two decision epochs, given the system occupies state $s$ and the decision maker chooses action $a$ in state $s$. This cost function is further discussed in Section II.C.

Since the optimization problem that we consider is to minimize the expected total cost, we define that a policy $\pi^*$ is optimal in $\Pi$ if $v^{\pi^*}(s) \leq v^{\pi}(s)$ for all $\pi \in \Pi$.

Let $G(t|X_n, Y_n)$ denote the cumulative distribution function of the time between decision epochs $n$ and $n+1$, given current state $X_n$ and action $Y_n$ is chosen. The time between decision epochs corresponds to the time between inter-switch handoffs. In this formulation, the time between inter-switch handoffs follows a general distribution and can depend on the location of a particular anchor switch that the mobile terminal is connected to. We use $G(dt|X_n, Y_n)$ to represent the time-differential. That is, $G(dt|X_n, Y_n) = dG(t|X_n, Y_n)$.

A policy is said to be stationary if $\delta_t = \delta$ for all $t$. A stationary policy has the form $\pi = (\delta, \delta, \cdots)$; for convenience we denote it by $\delta$. For a stationary policy $\delta$, (2) can be written as:

$$v^\delta(s) = c[s, \delta(s)] + E^\delta \left\{ e^{-\mu\tau_{\delta}} v^\delta(X_1) \right\}$$

where $\tau_{\delta}$ is the time spent between switching, and $\delta(s)$ is the stationary deterministic policy that minimizes (4).

To simplify the analysis, two assumptions are made. First, we assume the distribution of the time between inter-switch handoffs is independent of the state and action chosen, i.e., $G(dt|X_n, Y_n) = G(t)$. Second, we assume the mobile terminal is communicating with a remote terminal which is stationary. That is, we consider a mobile-to-fixed connection. The model formulation for mobile-to-mobile connection is described in Section V.A.
B. State Transition Probability Function

A state change occurs when there is an inter-switch hand-off. The state space $S$ is three dimensional. For each state $(i,j,k) \in S$, $i$ denotes the location of the target switch; $j$ denotes the location of the current anchor switch; and $k$ denotes the number of links of the current path. Thus,

$$S = \{1, 2, \ldots, N\} \times \{1, 2, \ldots, N\} \times \{1, 2, \ldots, L\}$$

where $N$ denotes the total number of nodes in the network and $L$ represents the maximum number of links allowed in a path. The number of links of any path is always finite. We assume the number of links increased by a path extension is bounded by $M$ which is much smaller than $L$ (i.e., $M \ll L$).

Since the end-to-end delay is proportional to the number of links of the path, a sub-optimal path with a large number of links not only increases the delay but also increases the call dropping probability and the congestion level of the network. We impose the condition that whenever the number of links in a connection is greater than or equal to $L - M$ and there is an inter-switch handoff, path extension is performed followed by path optimization with certainty. For convenience, let $K = L - M$. Later we show that path optimization is always performed when the number of links exceeds a certain threshold, and this threshold is much smaller than $K$.

Given the current state $(i,j,k)$, the available action set is:

$$A_{[i,j,k]} = \begin{cases} 
\{\text{NPO}, \text{PO}\}, & 1 \leq k < K \\
\{\text{PO}\}, & K \leq k \leq L.
\end{cases}$$

Thus, after each path extension, path optimization may be performed if the number of links is less than $K$, while path optimization is performed with certainty whenever the number of links is greater than or equal to $K$.

Two probability distribution functions are introduced to govern the state changes. Let

- $p(m[i], j)$ denote the probability that the number of links of the optimal path is $m$, given that the locations of the two endpoints are $i$ and $j$, respectively.
- $q(l|i)$ denote the probability that the location of the target switch in the next decision epoch is $l$, given that the location of the target switch in the current decision epoch is $i$.

In ATM networks, source routing is being used for all connection setup requests. That is, the source switch selects a path based on topology, loading, and reachability information in its database. As networks grow in size and complexity, full knowledge of network parameters is typically unavailable. Each single entity in the network cannot be expected to have detailed and instantaneous access to all nodes and links. Routing must rely on partial or approximate information, and still meet the QoS demands [15]. The ATM Forum PNNI standard [16] introduces a hierarchical process that aggregates information as the network gets more and more remote. However, the aggregation process inherently decreases the accuracy of the information and introduces uncertainty. Thus, in large networks it is more appropriate to model the number of links of a path between two endpoints in a probabilistic manner.

On the other hand, for small networks with periodic routing information updates, the number of links of a path between two endpoints can be modelled in a deterministic manner. Let $\Upsilon(i,j)$ denote the number of links of the optimal path between the two endpoints $i$ and $j$. The functions $p(m[i], j)$ and $\Upsilon(i,j)$ are related by

$$p(m[i], j) = \begin{cases} 
1, & \text{if } \Upsilon(i,j) = m \\
0, & \text{if } \Upsilon(i,j) \neq m.
\end{cases}$$

Let $D$ denote the location of the destination (i.e., the remote terminal), which is assumed to be fixed. The transition probability that the next state is $s' = (i', j', k')$ given that the current state is $s = (i,j,k)$ and action $a$ is chosen, is given by:

$$P(i', j', k'|i, j, k, a) = \begin{cases} 
q(i'|i)p(m[i], j), & i' \neq i, j' \neq i, k' = k + m, \\
q(i'|i)p(n[i], D), & i' \neq i, j' \neq i, k' = n, a = PO, \\
0, & \text{otherwise}.
\end{cases}$$

Equation (7) states that if action $\text{NPO}$ is chosen, the number of links is increased by $m$ with probability $p(m[i], j)$ after path extension. On the other hand, if action $\text{PO}$ is chosen, the number of links is equal to $n$ with probability $p(n[i], D)$ after path optimization. In both cases, the location of the target switch at the next decision epoch is equal to $i'$ with probability $q(i'|i)$.

C. Cost Functions

For each path extension event, the network incurs a fixed signaling cost $C_{PE} > 0$ and a variable signaling cost $h_{PE}(m)$ where $m$ represents the number of links increased during path extension. The terms $C_{PE}$ and $h_{PE}(m)$ capture the cost of setting up the extended path between the anchor and target switches.

For each path optimization performed, the network incurs a fixed signaling cost $C_{PO} > 0$ and a variable signaling cost $h_{PO}(l)$ where $l$ represents the number of links reduced during path optimization. These two terms capture the cost of (1) locating the crossover switch; (2) setting up the new branch connection; (3) terminating the old branch connection; and (4) updating the connection server about the status of the existing route.

We assume the link cost rate only depends on the number of links of the current path. That is, $f(s) = f(k)$ for all $s \in S$. Recall from (2) that $c(i,j,k,a)$ denotes the expected total cost between two decision epochs, given the system occupies state $(i,j,k)$ and action $a$ is chosen. Since the first inter-switch handoff occurs at time $\sigma_1$, the locations of the anchor and target switches are the same at the call setup time $\sigma_0$. Thus, during the time interval $[\sigma_0, \sigma_1]$, we have $i = j$ and the cost function

$$c(j,j,k, NPO) = I_j f(k)$$

(8)
where $I_1 = \int_0^\infty \int_0^\infty e^{-rt} drG(dt)$. The function $I_1 f(k)$ is the expected discounted link cost between two decision epochs, given that the current number of links is $k$.

For other decision epochs not equal to $\sigma_0$, the locations of the anchor and target switches are always different (i.e., $i \neq j$ if $\sigma \neq \sigma_0$). In that case, if action $NPO$ is chosen, then the cost function

$$ c(i, j, k, NPO) = C_{PE} + \sum_{m=1}^{M} [h_{PE}(m) + I_1 f(k + m)] \times p(m|i, j). $$

(9)

The function $C_{PE} + \sum h_{PE}(m)p(m|i, j)$ is the expected signaling cost for path extension, given that the locations of the anchor and target switches are $i$ and $j$, respectively.

For path optimization, we assume the number of links of the optimal path is always less than or equal to the number of links of the current path, and less than $K$. For decision epoch $\sigma$ not equal to $\sigma_0$, if action $PO$ is chosen, then the cost function

$$ c(i, j, k, PO) = C_{PE} + \sum_{m=1}^{M} h_{PE}(m)p(m|i, j) + C_{PO} + \sum_{m=1}^{M} \sum_{n=1}^{M} h_{PO}(k + m - n) + I_1 f(n)] \times p(n|i, D)p(m|i, j) $$

(10)

where $x \land y = \min(x, y)$. The expression $C_{PO} + \sum h_{PO}(k - n)p(n|i, D)$ is the expected signaling cost for path optimization, given that the current number of links is $k$, and the locations of the source and destination are $i$ and $D$, respectively.

### III. Optimality Equations

In this section, we introduce the optimality equations and investigate their properties. We show that solutions of these equations correspond to optimal value functions and that they also provide a basis for determining optimal policies. Let $v(s)$ denote the minimum expected total cost per call given state $s$. That is,

$$ v(s) = \min_{a \in A} v^\pi(s). $$

(11)

The optimality equations are given by

$$ v(s) = \min_{a \in A} \left\{ c(s, a) + \sum_{s' \in S} \left[ \int_0^\infty e^{-\mu t} v(s') P(s'|s, a)G(dt) \right] \right\} $$

(12)

Let $I_2 = \int_0^\infty e^{-\mu t} G(dt)$. Equation (12) can be expanded as follows: For $i = j$ and $1 \leq k < K$,

$$ v(j, j, k) = c(j, j, k, NPO) + \sum_{l=1}^{N} I_2 v(l, j, k)q(l|j). $$

(13)

For $i \neq j$ and $1 \leq k < K$,

$$ v(i, j, k) = \min \left\{ c(i, j, k, NPO) + \sum_{l=1}^{N} \sum_{m=1}^{M} I_2 v(l, i, k + m) \times p(m|i, j)q(l|i), c(i, j, k, PO) + \sum_{l=1}^{N} \sum_{m=1}^{M} \sum_{(k+m) \land (K-1)} I_2 v(n|i, D)p(m|i, j)q(l|i) \right\}. $$

(14)

For $i \neq j$ and $K \leq k \leq L$,

$$ v(i, j, k) = c(i, j, k, PO) + \sum_{l=1}^{N} \sum_{n=1}^{K-1} I_2 v(l, i, n)p(n|i, D)q(l|i). $$

(15)

At call setup time $\sigma_0$, the locations of the anchor and target switches are the same. Thus in (13), no path extension or path optimization is performed. For other decision epochs not equal to $\sigma_0$, the locations of the anchor and target switches are different. If the number of links of the path is less than $K$, then after each path extension, the network will decide whether to perform subsequent path optimization. This fact is stated in (14). Since path optimization is always performed if the number of links is greater than or equal to $K$, in (15), the action $PO$ is chosen when there is an inter-switch handoff.

If the signaling cost function for path optimization is zero (i.e., $C_{PO} = h_{PO}(l) = 0$), the problem of finding an optimal policy is trivial. It is optimal to perform path optimization after each inter-switch handoff. This is because the link cost function is nondecreasing with respect to the number of links of the current path. After each path optimization, there is a reduction in the number of links. However, if the signaling cost function for path optimization is nonzero, it is not obvious as to what constitutes the optimal policy. Note that if $\mu > 0$, the state space is finite, and the cost functions are bounded, then the solutions for equations (13)-(15) exist. By solving these equations, a stationary deterministic optimal policy can be obtained.

#### A. Value Iteration Algorithm

There are a number of iteration algorithms available to solve the above optimality equations. Examples include the value iteration, policy iteration, and linear programming algorithms [13]. Value iteration is the most widely used and best understood algorithm for solving discounted Markov decision problems. The following value iteration algorithm finds a stationary deterministic optimal policy and the corresponding expected total cost.

**Algorithm**

1. Set $v^0(s) = 0$ for each state $s \in S$. Specify $\epsilon > 0$ and set $n = 0$.
2. For each $s \in S$, compute $v^{n+1}(s)$ by:

$$ v^{n+1}(s) = \min_{a \in A} \left\{ c(s, a) + \sum_{s' \in S} \left[ \int_0^\infty e^{-\mu t} v^{n}(s') P(s'|s, a)G(dt) \right] \right\}. $$

(16)
3. If \( \|v^{n+1} - v^n\| < \epsilon \), go to step 4. Otherwise increment \( n \) by 1 and return to step 2.

4. For each \( s \in S \), compute the stationary optimal policy

\[
\delta(s) = \arg \min_{a \in A} \left\{ c(s, a) + \sum_{s' \in S} P(s'|s, a) G(dt) \right\}
\]

and stop.

In this paper, the function norm is defined as: \( \|v\| = \max_{s \in S} v(s) \). Convergence of the value iteration algorithm is ensured since the operation in step 2 corresponds to a contraction mapping. Thus, the function \( v^n(s) \) converges in norm to \( v(s) \). Note that the convergence rate of the value iteration algorithm is linear.

In small networks, if each node maintains perfect information of all nodes and links, then the function \( v[i, j, \mathcal{Y}(i, D)] \) is the minimum expected total cost per call given source \( i \) and destination \( D \). On the other hand, in large networks, the number of links of a path determined by the source is modeled in a probabilistic manner. In that case, the expression

\[
\sum_k v(i, j, k) p(k|i, D)
\]

is the minimum expected total cost per call given source \( i \) and destination \( D \), averaged over the number of links of the optimal path.

### B. Structure of the Optimal Policy

We now provide a condition under which the optimal policy has a control limit (or threshold) structure. The control limit structure states that path optimization is performed with certainty whenever the number of links of the current path exceeds a certain threshold. For convenience, we let \( \Delta \gamma(k) = \gamma(k + 1) - \gamma(k) \) for some function \( \gamma \).

**Proposition 1:** Given state \( (i, j, k) \in S \), there exists an optimal policy \( \delta^* \) that has a control limit structure:

\[
\delta^*(i, j, k) = \begin{cases} 
NPO, & 1 \leq k < k^* \\
PO, & k^* \leq k \leq L 
\end{cases}
\]

when \( I_1 \Delta f(k + m) - \sum_n \Delta h_{P0}(k + m - n)p(n|i, D) \geq 0 \) for all \( m \) such that \( p(m|i, j) \neq 0 \) and \( k^* \leq k < K \).

The proof of the above proposition is shown in the appendix. The value \( k^* \) is the control limit or threshold. Consider the special case where the cost functions are linear. That is, \( f(k) = C_{link} \cdot k \) and \( h_{P0}(l) = w_{P0} \cdot l \) where \( C_{link} \) and \( w_{P0} \) are positive constant. In this case, if \( I_1 C_{link} - w_{P0} \geq 0 \), then path optimization is always performed when the number of links is greater than or equal to \( k^* \). An optimal policy with threshold structure facilitates its implementation. For each mobile connection, the network only has to maintain the information of the minimum number of links to initiate path optimization for all anchor and target switch pairs. The decision to perform path optimization can be made via a table lookup.

Note that the optimal policy still maintains a threshold structure for other cost functions as long as they are convex and nondecreasing. Interested readers can refer to [19] for a proof of this fact. For those cost functions, the value iteration algorithm can still be used to determine the minimum expected total cost and the optimal policy.

### IV. Implementation Aspects

Having identified the different parameters involved in the model, we are now in a position to explain the steps that need to be taken in order to implement the model. For each mobile connection, during its connection setup phase, the network controller assigns the cost functions based on the service class and the signaling load of the network. Different service classes with different bandwidth requirements are assigned different link cost functions to reflect the network resources consumed. The assigned signaling cost function reflects the complexity of the path optimization procedures and the current signaling load of the network. By keeping the mobility profile of each user (i.e., the movement history and call history), the average time between inter-switch handoff as well as the average duration of the connection can be estimated [18][19].

Given the input parameters (i.e., cost functions and various distributions), the value iteration algorithm can be used to determine the optimal policy. The optimal policy is then stored in a tabular format. Each entry of the table specifies the minimum number of links to initiate path optimization for a specific pair of anchor and target switches. Whenever there is an inter-switch handoff, the network performs a table lookup at the corresponding anchor and target switch entry. Path optimization is performed if the number of links is greater than the threshold. The optimal policy table needs to be updated when there are changes in network topology or signaling load of the network. The update can be performed off-line. That is, whenever spare processing capacity is available at the network controller.

### V. Model Extensions

In the previous sections, we consider a connection between a mobile terminal and a fixed endpoint. In this section we extend the model to a connection between two mobile terminals and take into consideration other QoS constraints.

#### A. Extension to Mobile-to-Mobile Connection

The problem formulation for mobile-to-mobile connection is similar to that of mobile-to-fixed connection. Consider mobile terminals 1 and 2 communicating with each other via a wireless ATM network. Each mobile terminal has its own movement pattern. A path extension is performed when there is an inter-switch handoff, (initiated from either side), followed by a path optimization if necessary. In this formulation, the state space needs to include the locations of the two endpoints, as well as the information of which mobile terminal initiates the path extension. For each state \( (i, i_1, j_2, k, \theta) \in S \), \( i \) denotes the location of the target switch; \( j_1 \) and \( j_2 \) denote the locations of the
anchor switches connected to mobile terminals 1 and 2, respectively; \( k \) denotes the number of links of the current path; and \( \vartheta \) denotes the identifier of the mobile terminal which initiates the inter-switch handoff.

Since the movement pattern of each mobile user is different, the time between inter-switch handoffs for each mobile user is also different. Suppose the time between inter-switch handoffs for mobile terminal \( r \), \( (r \in \{1, 2\}) \), is exponentially distributed with rate \( \lambda_r \), then the time between decision epochs is also exponentially distributed with rate \( (\lambda_1 + \lambda_2) \).

Since the state space has changed, the cost functions and the state transition probability function have to be modified accordingly. As the modification is conceptually similar to the functions derived in Section II, the details are omitted. The optimality equations are

\[
v(s) = \min_{a \in A} \left\{ c(s,a) + \left( \frac{\lambda_1 + \lambda_2}{\lambda_1 + \lambda_2 + \mu} \right) \sum_{s' \in S} [v(s')]P(s'|s,a) \right\}.
\]

The value iteration algorithm can be used to evaluate the expected total cost and the optimal policy. The conditions for the optimal policy with a threshold structure can also be derived.

B. Extension to QoS Constraints

In Sections II and III, path optimization is triggered based on the number of links of the current path. In general, a mobile connection can have multiple QoS constraints such as bandwidth, delay, delay jitter, etc. Suppose the connection has to maintain a delay constraint. In this case path optimization is performed with certainty if the end-to-end delay after path extension exceeds the delay constraint, while path optimization may be performed if the delay after path extension is still below the constraint.

To incorporate the delay constraint into the model, the state space needs to be extended to include the end-to-end delay of the current path. We assume the end-to-end delay of a path is the sum of the delay on each link of the path. The delay information on each link can be obtained from the network by measurement. Let \( \zeta \) denote the end-to-end delay of the current path and \( \Psi \) be the delay constraint. Let \( \Phi(i,j) \) denote the delay of the path between the two endpoints \( i \) and \( j \). The optimality equations described in Section III then include the constraint \( \zeta + \Phi(i,j) \leq \Psi \) where \( i \) and \( j \) denote the locations of target and anchor switches respectively. Note that the value iteration algorithm cannot be used to solve the optimality equations with constraints. However, the optimality equations can be transformed into primal or dual linear programs, which can then be solved by the simplex algorithm. Due to the space limitation, please refer to [13] for the details of the transformation.

In summary, multiple QoS constraints can be incorporated into the model by extending the state space and including the constraint equations into the set of optimality equations. The expected total cost and the optimal policy can be obtained by transforming the model into a linear programming model.

VI. NUMERICAL RESULTS AND DISCUSSIONS

In this section, we compare the performance of the optimal policy with four heuristics. For the first heuristic, path optimization is performed after each path extension. We denote this policy as “always perform PO” or \( \delta^{PO} \). For the second heuristic, no path optimization is performed during the connection lifetime. We denote this policy as “never perform PO” or \( \delta^{NPO} \). For the third heuristic, periodic path optimization [20] is considered. For periodic path optimization, after each fixed time period, the network determines if the connection requires path optimization. Path optimization is performed if an inter-switch handoff has occurred during the time interval. We assume this fixed time interval is equal to the average time between inter-switch handoffs. For the last heuristic, we consider the Bernoulli path optimization scheme which we proposed and analyzed in [21]. For the Bernoulli scheme, path optimization is performed with probability \( p_{opt} \) after each extension.

The performance metrics are the expected total cost per call and the expected number of path optimizations per call. The expected total cost per call is defined in Section II. The expected number of path optimizations per call given policy \( \pi \) with initial state \( s \) is:

\[
\gamma_\pi(s) = E^\pi_s \left\{ \sum_{n=0}^{\infty} e^{-\mu \tau_n} \times 1[Y_n = 1] \right\}
\]

where \( 1[\cdot] \) denotes the indicator function (i.e., \( 1[a = 1] \) is equal to 1 if \( a = 1 \) and 0 otherwise).

A. Simulation Model

In the simulation model, a wireless ATM network is modelled as a non-hierarchical random graph. Random graphs have been used to model ATM networks [22]. Different variations of random graph models have also been proposed to model the topology of the Internet [23][24]. The generation of a non-hierarchical random graph consists of the following steps [23]:

1. \( N \) nodes are randomly distributed over a rectangular coordinate grid. Each node is placed at a location with integer coordinates. A minimum distance is specified so that a node is rejected if it is too close to another node. The Euclidean metric is used to calculate the distance \( \alpha(i,j) \) between each pair of nodes \( (i,j) \).
2. A fully connected graph is constructed with the link weight equal to the Euclidean distance.
3. Based on the fully connected graph, a minimum weight spanning tree is constructed.
4. To achieve a specified average node degree of the graph, edges are added one at a time with increasing distance. If node \( i \) and \( j \) are connected, then the link weight, denoted as \( \omega_{ij} \), is assumed to be equal to:

\[
\omega_{ij} = \alpha(i,j) + \beta
\]
where $\beta$ is a uniformly distributed random variable in the range $0 \leq \beta \leq \beta_{\text{max}}$. In (19), the first term can be interpreted as the propagation delay of the link, and the second term approximately models the queuing delay of the link.

Figure 4 shows a 20-node random graph with an average node degree of 3. The minimum distance between any two nodes is 15. The value of $\beta_{\text{max}}$ is 100. Each node represents an ATM switch and each edge represents a physical link connecting two switches. Since we are only concerned about inter-switch handoff, base stations are not included in the model.

Based on the above network model, we obtain the adjacency matrix of the network as well as the number of links of the shortest path between any two nodes. We assume the number of links of the shortest path estimated by the source is deterministic. The call duration is assumed to be exponential. The time between inter-switch handoffs follows a Gamma distribution. When there is an inter-switch handoff, we assume each of the neighboring switches has the same probability to be the target switch.

For each source and destination pair, the value iteration algorithm is used to determine the minimum expected total cost and the optimal policy. For the value iteration algorithm, $\epsilon$ is chosen to be equal to $10^{-8}$. From the optimal policy, the value iteration algorithm is used again to calculate the expected number of path optimizations by solving (18). The minimum expected total cost and the expected number of path optimizations are then averaged over all possible source and destination pairs. We repeat this for 100 random graphs and determine the averages.

For the two heuristic policies $\delta_{\text{PO}}$ and $\delta_{\text{NPO}}$, the expected total cost and the expected number of path optimizations for each source and destination pair are also determined by the value iteration algorithm. These values are then averaged over all possible source and destination pairs. Again, we repeat this for 100 random graphs and determine the averages.

For the periodic and Bernoulli path optimization policies, simulation must be used. Given the network topology, a call is generated with two nodes chosen as the source and destination. Dijkstra’s algorithm is used to compute the shortest path between these two nodes. The destination node is assumed to be stationary. The source node becomes the anchor switch of the mobile connection. During each inter-switch handoff, the target switch is restricted to be one of the neighboring switches of the current anchor switch. Path extension is used to extend the connection from the anchor switch to the target switch. Path optimization is performed periodically for the periodic scheme. For the Bernoulli scheme, path optimization is performed with probability $p_{\text{opt}}$ after each extension. For each source and destination pair, 1000 simulation runs are performed. The average total cost and the average number of path optimizations per call are determined. We repeat this for 100 random graphs and determine the averages.

All the cost functions are assumed to be linear. The link cost function $f(k) = \gamma_{\text{link}} \cdot k$ where $\gamma_{\text{link}} > 0$. The term $\gamma_{\text{link}}$ captures the bandwidth used by the connection. Different $\gamma_{\text{link}}$ can be assigned for different traffic classes.

The variable cost function for path extension $\eta_{\text{FE}}(m) = w_{\text{FE}} \cdot m$ where $w_{\text{FE}} > 0$ and $m$ denotes the number of links increased during path extension. The variable cost function for path optimization $\eta_{\text{PO}}(l) = w_{\text{PO}} \cdot l$ where $w_{\text{PO}} > 0$ and $l$ denotes the number of links reduced during path optimization.

**B. Results**

Figure 5 shows the expected total cost versus the link cost rate $\gamma_{\text{link}}$. The optimal policy gives the lowest expected total cost compared to the other four heuristics. When $\gamma_{\text{link}}$ is small, there is no incentive to perform path optimization. The operating point, $p_{\text{opt}}$, for the Bernoulli policy is close to zero. The optimal policy is to perform path extension only. Thus, results of the Bernoulli, $\delta_{\text{NPO}}$, and optimal policies are the same. When $\gamma_{\text{link}}$ increases, the optimal policy for some source and destination pairs is to perform path optimization. Results of the optimal, Bernoulli, and $\delta_{\text{NPO}}$ policies diverge, while the results of the Bernoulli and $\delta_{\text{PO}}$ policies begin to converge.

Figure 6 shows the expected number of path optimizations versus $\gamma_{\text{link}}$. Since no path optimization is performed for the $\delta_{\text{NPO}}$ policy, the expected number of path optimizations is always equal to zero. Note that since both the call termination rate and the inter-switch handoff rate are constant, in this case the expected number of inter-switch handoffs is also a constant. Thus, results for the periodic and $\delta_{\text{PO}}$ policies are independent of $\gamma_{\text{link}}$. For the Bernoulli and optimal policies, when $\gamma_{\text{link}}$ is small, there is no incentive to perform path optimization. The expected number of path optimizations is small. As $\gamma_{\text{link}}$ increases, some source and destination pairs perform path optimization after inter-switch handoff. Thus, there is an increase in the number of path optimizations performed.

Figure 7 shows the expected total cost versus the inter-switch handoff rate $\lambda$. The expected total cost increases as $\lambda$ increases. When $\lambda$ is small (i.e., the average time between inter-switch handoffs is larger than the average call duration), an inter-switch handoff is unlikely to occur during the connection lifetime. Thus, the results between the five policies are close. As $\lambda$ increases, these five curves begin to diverge. The $\delta_{\text{PO}}$ policy gives the highest expected total cost, which is followed by the periodic, $\delta_{\text{NPO}}$, and Bernoulli policies. Results of the $\delta_{\text{NPO}}$ and Bernoulli policies are very close. Although we can conclude that the expected total cost increases in $\lambda$ and the optimal policy always gives the minimum expected total cost, the performance comparisons between the other four heuristics differ when another set of parameters are chosen. That is, the $\delta_{\text{PO}}$ policy can sometimes have a better performance than the periodic and $\delta_{\text{NPO}}$ policies.

Figure 8 shows the expected number of path optimizations versus $\lambda$. The expected number of path optimizations increases as $\lambda$ increases. Results of the Bernoulli and optimal policies are quite close. Due to the threshold structure of the optimal policy, path optimization is performed only after a certain number of inter-switch handoffs. Thus, the
expected number of path optimizations for the optimal policy is smaller than the periodic and $\delta^{PO}$ policies.

Figure 9 shows the expected total cost versus the call termination rate $\mu$. The expected total cost decreases as $\mu$ increases, which is intuitive since the link cost is accrued continuously during the call lifetime. When $\mu$ is large (i.e., the call duration is short), all the connections experience a small number of inter-switch handoffs. Thus, the results of all these policies are close. When the call duration increases, the results begin to diverge. We can see a significant cost difference between the optimal policy and the other heuristics when the call duration is long.

Figure 10 shows the expected number of path optimizations versus $\mu$. The expected number of path optimizations decreases as $\mu$ increases. Due to the threshold structure of the optimal policy, path optimization is performed only after a certain number of path extensions. Thus, the expected number of path optimizations performed for the optimal policy is much smaller than the periodic and $\delta^{PO}$ policies.

In the previous results, we assume the time between inter-switch handoffs follows a Gamma distribution. We also consider exponential and hyper-exponential distributions for the time between inter-switch handoffs. For a fair comparison, the average time between inter-switch handoffs is the same for various distributions. Figures 11 and 12 show the minimum expected total cost of the optimal policy versus $\lambda$ and $C_{in,k}$, respectively. These results indicate that the expected total cost is relatively insensitive to the distributions of the time between inter-switch handoffs.

C. Sensitivity Analysis

In order to calculate the minimum expected cost, the optimal policy table needs to be determined. The policy obtained depends on the values of different parameters (e.g., $\lambda$, $\mu$, $C_{in,k}$, and $C_{PO}$). Although the parameters $C_{in,k}$ and $C_{PO}$ can be determined by the network, the values of $\lambda$ and $\mu$ may not always be estimated correctly by the mobile terminal during call setup. If that is the case, the optimal policy may not indeed be the optimal one. In this section, we are interested in determining the percentage change of the expected cost per call to the variation of the average call duration and the average time between inter-switch handoffs. The procedures for the sensitivity analysis consist of the following steps:

1. Given the actual call termination rate $\mu$ and other cost and mobility parameters, we first determine the minimum expected total cost, denoted as $Cost\,(optimal)$.
2. Let $\hat{\mu}$ denote the estimated call termination rate and $\Delta_\mu$ denote the percentage change of the average call duration. These parameters are related by the following equation:

   \begin{equation}
   \hat{\mu}^{-1} = (1 + \Delta_\mu)\mu^{-1}.
   \end{equation}

Based on the estimated call duration rate $\hat{\mu}$ and other parameters, the sub-optimal policy is determined. From this sub-optimal policy and other cost and mobility parameters (i.e., $\lambda$, $\mu$, etc.), the sub-optimal expected total cost, denoted as $Cost\,(sub-optimal)$, is computed.

3. The change in the expected total cost with respect to the variation of the call duration is characterized by the cost ratio, which is defined as: $Cost\,(sub-optimal) / Cost\,(optimal)$.

The results for different $\mu$ are shown in Figure 13. When the average call duration is over-estimated by more than $-40\%$, the cost ratio is almost equal to one, which implies that the optimal policy is insensitive to the change of the average call duration. However, within the $(-90, -50)$ percentage range, there is an increase in the cost ratio. The cost ratio can be as high as 1.53 for $\mu = 0.02$. These results imply that if there is uncertainty in estimating the average call duration, it may be better to over-estimate the value in order to reduce the cost ratio difference.

We use the similar procedures described above to investigate the percentage change of the expected total cost to the variation of the time between inter-switch handoffs. Figure 14 shows the cost ratio versus the percentage change in average time between inter-switch handoffs for different $\lambda$. Within the percentage range of interest, the cost ratio is always less than 1.07 (i.e., 7%). Within the $(-50, 100)$ percentage range, the cost ratio is less than 1.01 (i.e., 1%). These results imply that the optimal policy is relatively insensitive to the change of the average time between inter-switch handoffs.

D. Discussions

In our simulation studies, we found the value iteration algorithm to be very efficient and stable. The number of iterations is quite predictable from point to point, changing slowly as the independent parameter changes. In general, the number of iterations to convergence does not depend on the cost parameters ($C_{in,k}$, $C_{PO}$, $C_{PE}$), but depends on the values of $\lambda$ and $\mu$. As an example, for the optimal policy in Figure 9, the value iteration algorithm required only 24 iterations to converge when $\mu = 0.1$, but it required 170 iterations when $\mu = 0.01$. Note that there are other iteration algorithms available (e.g., policy iteration algorithm) which have a higher rate of convergence. Interested readers can refer to [13] for details.

In this paper, the wireless ATM network is modelled as a non-hierarchical random graph. One question that arises is whether the results will differ if some other network topologies are being used. The answer is affirmative. The relative performance between the four heuristics will change if another network topology is being used. This is essentially the same as changing the values in the functions $\mu(m|c)$ or $\Upsilon(i,j)$. However, the optimal policy always gives the lowest expected total cost compared to the other four heuristics.

VII. Conclusions

In this paper, we have addressed the issue of when to initiate path optimization for the two-phase handoff protocol. The path optimization problem is formulated as a semi-Markov decision process. A link cost function is used to reflect the network resources utilized by a connection. Signaling cost function is used to capture the signaling and processing load incurred on the network. The time between
inter-switch handoffs follows an arbitrary general distribution. When an inter-switch handoff occurs, based on the current state information, the network controller decides whether to perform path optimization after path extension.

We have presented the value iteration algorithm which determines the expected total cost and the optimal policy. Under certain conditions, we have shown the existence of an optimal policy which has a threshold structure. That is, path optimization is always performed when the number of links of the path is greater than a certain threshold. The threshold structure of the optimal policy facilitates the implementation. When an inter-switch handoff occurs, the decision of performing path optimization can be made by a simple table lookup.

The performance of the optimal policy has been compared with four heuristics. Simulation results indicate that the optimal policy gives a lower expected cost per call than those heuristics. These results imply that by using the optimal policy, the mobile connection maintains a good balance between the network resources utilized and the signaling load incurred on the network during its connection lifetime. We have also performed sensitivity analysis for the optimal policy with respect to the variation of the average call duration and the average time between inter-switch handoffs. Results indicate that the optimal policy is relatively insensitive to the change of the average time between inter-switch handoffs. If there is uncertainty in estimating the average call duration, it may be better to over-estimate the value in order to reduce the cost ratio difference.

Future work includes extending the proposed model to analyze the (1) mobile-to-mobile connection scenario; (2) multicast connection in which a group of mobile users are communicating with each other; and (3) path optimization problem with QoS constraints. Although the proposed model captures the trade-off between the network resources used and the handoff processing and signaling load incurred on the network, the model is not without drawbacks. In our formulation, the call duration is exponentially distributed. Although the exponential distribution is valid for voice traffic, this may not be appropriate for multimedia applications. This also points to the need for new analytical models for general call durations.

**Appendix**

**Lemma 1:** For each state \((i, j, k) \in S\), the expected total cost \(v(i, j, k)\) is a nondecreasing function with respect to the number of links \(k\).

**Proof:** The proof of this lemma is by induction. We must show \(v(i, j, k - 1) \leq v(i, j, k + 1)\) for all \(k\). Hence,

\[
v(i, j, k) \\
\leq c(i, j, k + 1, PO) + \sum_{i=1}^{N} \sum_{n=1}^{K-1} I_2 v(l, i, n) p(n|i, D) q(l|i) \\
= v(i, j, k + 1).
\]

Thus, \(v(i, j, k) \leq v(i, j, k + 1)\) for \(K \leq k < L\). Since \((K - 1 + m) \wedge (K - 1) = K - 1\), for state \((i, j, K)\):

\[
v(i, j, K - 1) \\
= \min \left\{ \sum_{i=1}^{N} \sum_{m=1}^{M} I_2 v(l, i, K - 1 + m) p(m|i, j) q(l|i) \\
+ c(i, j, K - 1, NPO), c(i, j, K - 1, PO) + \sum_{i=1}^{N} \sum_{m=1}^{M} \sum_{n=1}^{K-1} I_2 v(l, i, n) p(n|i, D) p(m|i, j) q(l|i) \right\} \\
\leq c(i, j, K - 1, PO) + \sum_{i=1}^{N} \sum_{n=1}^{K-1} I_2 v(l, i, n) p(n|i, D) q(l|i) \\
\leq v(i, j, K).
\]

For \(i \neq j\) and \(1 \leq k < K\), assume \(v(i, j, k + 1) \leq v(i, j, k + 2) \leq \cdots \leq v(i, j, L)\). We need to show that \(v(i, j, k) - v(i, j, k + 1) \leq 0\). From (14),

\[
v(i, j, k) \\
= \min \left\{ \sum_{i=1}^{N} \sum_{m=1}^{M} I_2 v(l, i, k + m) p(m|i, j) q(l|i) \\
+ c(i, j, k, NPO), c(i, j, k, PO) + \sum_{i=1}^{N} \sum_{m=1}^{M} \sum_{(k+m) \wedge (K-1)} I_2 v(l, i, n) p(n|i, D) p(m|i, j) q(l|i) \right\}
\]

Let \(a^*\) denote the optimal action of state \((i, j, k + 1)\). If \(a^* = PO\),

\[
v(i, j, k) \\
\leq c(i, j, k, PO) + \sum_{i=1}^{N} \sum_{m=1}^{M} \sum_{n=1}^{K-1} I_2 v(l, i, n) \\
\times p(n|i, D) p(m|i, j) q(l|i) \\
\leq c(i, j, k + 1, PO) + \sum_{i=1}^{N} \sum_{m=1}^{M} \sum_{n=1}^{K-1} I_2 v(l, i, n) \\
\times p(n|i, D) p(m|i, j) q(l|i) \\
= v(i, j, k + 1).
\]
On the other hand, if $a^* = NPO$,

$$v(i, j, k) \leq c(i, j, k, NPO) + \sum_{l=1}^{N} \sum_{m=1}^{M} I_2v(l, i, k + m)p(m|i, j) \times q(l|i) \leq c(i, j, k + 1, NPO) + \sum_{l=1}^{N} \sum_{m=1}^{M} I_2v(l, i, k + 1 + m) \times p(m|i, j)q(l|i) = v(i, j, k + 1).$$

To complete the proof, we need to show $v(j, j, k) - v(j, j, k + 1) \leq 0$ for $1 \leq k < K$. From (13),

$$v(j, j, k) = c(j, j, k, NPO) + \sum_{l=1}^{N} I_2v(l, j, k)q(l|i) \leq c(j, j, k + 1, NPO) + \sum_{l=1}^{N} I_2v(l, j, k + 1)q(l|i) \leq v(j, j, k + 1).$$

Thus, by the principle of induction, for each state $(i, j, k) \in S$, the expected total cost $v(i, j, k)$ is a nondecreasing function in $k$. □

The proof of Proposition 1 is given below:

**Proof:** Let

$$r(i, j, k) = \sum_{m=1}^{M} I_1f(k + m)p(m|i, j) - C_{PO} + \sum_{l=1}^{N} \sum_{m=1}^{M} I_2v(l, i, k + m)p(m|i, j)q(l|i) - \sum_{m=1}^{M} \sum_{n=1}^{(k+m) \land (K-1)} \{[h_{PO}(k + m - n) + I_1f(n)]p(n|i, D) \times p(m|i, j)q(l|i)\} \sum_{l=1}^{N} \sum_{m=1}^{(k+m) \land (K-1)} I_2v(l, i, n)p(n|i, D)p(m|i, j)q(l|i).$$

Thus, the action $PO$ is chosen if $r(i, j, k) \geq 0$ and the action $NPO$ is chosen if $r(i, j, k) < 0$. Let $\hat{k}$ be the smallest $k$ such $r(i, j, k) \geq 0$. For convenience, let $\Delta r(i, j, k) = r(i, j, k + 1) - r(i, j, k)$. Then

$$\Delta r(i, j, k) = \sum_{m=1}^{M} I_1\Delta f(\hat{k} + m)p(m|i, j) \sum_{l=1}^{N} \sum_{m=1}^{M} I_2\Delta v(l, i, \hat{k} + m)p(m|i, j)q(l|i) - \sum_{m=1}^{M} \sum_{n=1}^{(k+m) \land (K-1)} \Delta h_{PO}(k + m - n)p(n|i, D)p(m|i, j).$$

Since $v(i, j, k)$ is a nondecreasing function in $k$, $\Delta v(l, i, k + m) \geq 0$. Thus, $\Delta r(i, j, k) \geq 0$ when

$$I_1\Delta f(\hat{k} + m) - \sum_{n=1}^{(k+m) \land (K-1)} \Delta h_{PO}(k + m - n)p(n|i, D) \geq 0$$

for all $m$ such that $p(m|i, j) \neq 0$. Now for some $k$, assume $\Delta r(i, j, k), \Delta r(i, j, k + 1), \ldots, \Delta r(i, j, k - 1) \geq 0$. Then

$$\Delta r(i, j, k) = \sum_{l=1}^{N} \sum_{m=1}^{M} I_2\Delta v(l, i, k + m)p(m|i, j)q(l|i) + \sum_{m=1}^{M} I_1\Delta f(k + m)p(m|i, j) - \sum_{m=1}^{M} \sum_{n=1}^{(k+m) \land (K-1)} \Delta h_{PO}(k + m - n)p(n|i, D)p(m|i, j).$$

Since $v(i, j, k)$ is a nondecreasing function in $k$, $\Delta v(l, i, k + m) \geq 0$. Thus, $\Delta r(i, j, k) \geq 0$ when

$$I_1\Delta f(k + m) - \sum_{n=1}^{(k+m) \land (K-1)} \Delta h_{PO}(k + m - n)p(n|i, D) \geq 0$$

for all $m$ such that $p(m|i, j) \neq 0$. Thus, by induction, the optimal policy has a threshold structure when $I_1\Delta f(k + m) - \sum_{n=1}^{M} \Delta h_{PO}(k + m - n)p(n|i, D) \geq 0$ for all $m$ such that $p(m|i, j) \neq 0$ and $k^* < k < K$. □

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**References**


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Fig. 1. (a) Path extension scheme; (b) Path rerouting scheme.

Fig. 2. Two-phase handoff protocol.

Fig. 3. Timing diagram.

Fig. 4. A 20-node random graph with average node degree of 3.

Fig. 5. Expected total cost versus link cost rate $C_{\text{link}} \ (\lambda = 0.1, \mu = 0.03, C_{PE} = 1, C_{PO} = 5, w_{PE} = w_{PO} = 0.5)$.

Fig. 6. Expected number of path optimizations versus link cost rate $C_{\text{link}} \ (\lambda = 0.1, \mu = 0.03, C_{PE} = 1, C_{PO} = 5, w_{PE} = w_{PO} = 0.5)$.
Fig. 7. Expected total cost versus inter-switch handoff rate $\lambda (\mu = 0.03, C_{\text{link}} = 0.1, C_{P_E} = 1, C_{P_O} = 5, w_{P_E} = w_{P_O} = 0.5)$.

Fig. 8. Expected number of path optimizations versus inter-switch handoff rate $\lambda (\mu = 0.03, C_{\text{link}} = 0.1, C_{P_E} = 1, C_{P_O} = 5, w_{P_E} = w_{P_O} = 0.5)$.

Fig. 9. Expected total cost versus call termination rate $\mu (\lambda = 0.1, C_{\text{link}} = 0.1, C_{P_E} = 1, C_{P_O} = 5, w_{P_E} = w_{P_O} = 0.5)$.

Fig. 10. Expected number of path optimizations versus call termination rate $\mu (\lambda = 0.1, C_{\text{link}} = 0.1, C_{P_E} = 1, C_{P_O} = 5, w_{P_E} = w_{P_O} = 0.5)$.

Fig. 11. Minimum expected total cost of the optimal policy versus inter-switch handoff rate $\lambda$ for various distributions ($\mu = 0.03, C_{\text{link}} = 0.1, C_{P_E} = 1, C_{P_O} = 5, w_{P_E} = w_{P_O} = 0.5$).

Fig. 12. Minimum expected total cost versus link cost rate $C_{\text{link}}$ for various inter-switch handoff rate distributions ($\lambda = 0.1, \mu = 0.03, C_{P_E} = 1, C_{P_O} = 5, w_{P_E} = w_{P_O} = 0.5$).
Fig. 13. Variation of the average call termination time ($\lambda = 0.1$, $C_{\text{link}} = 0.1$, $C_{\text{PE}} = 1$, $C_{\text{PO}} = 5$, $w_{\text{PE}} = w_{\text{PO}} = 0.5$).

Fig. 14. Variation of the average time between inter-switch handoffs ($\mu = 0.03$, $C_{\text{link}} = 0.1$, $C_{\text{PE}} = 1$, $C_{\text{PO}} = 5$, $w_{\text{PE}} = w_{\text{PO}} = 0.5$).