Motivating Problems
- What is the best staffing level in a hospital?
- What is the best admission control policy in a call center?
- What is the best ambulance positioning design in a city?

Introduction
More generally, we have a finite number \( k \) of alternative systems whose performance cannot be evaluated directly.
- We use a stochastic simulator to evaluate the mean performance \( \mu_x \) for each \( x \) with noise. However, simulation is expensive, and our sampling budget is limited.
- Our goal is to find \( x^* \approx \arg\max_x \mu_x \).

How to Allocate Simulation Effort Efficiently?
- We use a Bayesian value-of-information (VOI) approach (Howard 1966, Chick & Gans 2009, Chick & Frazier 2009) to design sampling strategies. They are called
  - or knowledge-gradient (KG) policies (Frazier 2009).

Basic Idea:
Given the function evaluations obtained, use Bayesian statistics (e.g., Gaussian Process (GP) regression) to get:
- estimates of \( \mu_x \) and uncertainties in these estimates.
- Then, use these estimates to make decisions to
  - assess the VOI of possible future evaluations.
  - decide where to evaluate next.

Our Novelty:
- Allow for correlation in prior beliefs on \( \theta \) and across samples.
- Allow for sampling from a prior in each given stage.
- Handle large-scale problems with many alternatives.

Modeling
Our belief about the performance of the alternatives \( \theta \) is a Gaussian process prior over a discrete set, and thus is a correlated multivariate normal prior, i.e., \( \theta \sim \mathcal{N}(\mu, \Sigma) \).

We allowed correlated simulation output through common random numbers (CRN), and observe \( Y \sim X(\theta, \phi) \) if sampling all alternatives together using CRN.
At each time \( n \) we:
- calculate the Bayesian posterior distribution on \( \theta \) given what we currently know, which is \( X(n; \mu_n, \Sigma_n) \).
- choose a subset of alternatives \( X_{n+1} \) to sample at time \( n+1 \).

If \( n = N \) (stopping time), report \( \hat{x}(n) \approx \arg\max_x \mu_x \) as the "implementation decision", else increment \( n \).

Value of Information
- At time \( n \), the VOI of choosing a subset of alternatives \( X \) (cf. Ginsbourger et al.) to evaluate at time \( n+1 \) is
  \[ V(\theta)(X) = \max_{x \in X} \mathbb{E}[\mu_x + \log f(y)] - \max_{x \in X} \mathbb{E}[\mu_x] \]
- \( \max_{x \in X} \mathbb{E}[\mu_x] \) is the expected value of \( \mu_x \) at time \( n \).
- \( \max_{x \in X} \mathbb{E}[\mu_x] \) is the best we can do at time \( n \).
- \( \max_{x \in X} \mathbb{E}[\mu_x] \) is the best we will be able to do after seeing \( X_{n+1} \).
- \( V(\theta)(X) \) is a value of information (VOI).
- \( V(\theta)(X) \) is similar to the expected improvement (EI).
- \( V(\theta)(X) \) differs from EI (as usually defined) because it considers the change in the posterior mean at points that are not sampled.
- Frazier et al. 2011 gives analytical computation for
  - \( X = \{ \text{single} \} \)
  - \( X = \{ \text{pair} \} \) (observe the difference between a pair)

Conclusion
Our KG factor algorithm takes advantage of two types of correlations, deals with large-scale problems, and significantly reduces the number of samples required to find a good alternative.

References

Bayesian Optimization via Simulation with Pairwise Sampling & Correlated Beliefs
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