The problem we are interested in is the following:

\[
\begin{align*}
\max & \sum_{i \in I} \sum_{j \in F} c_{ij} x_{ij} + \mathbb{E}Q(s, D) \\
\text{subject to:} & \sum_{j \in F} x_{ij} \leq p_i, \quad i \in I, \\
& \sum_{i,j \in F} x_{ij} - s_j = 0, \quad j \in I \cup C, \\
& x_{ij}, s_j \geq 0,
\end{align*}
\]

where \( Q(s, D) \) is the optimal value of the second stage problem:

\[
\begin{align*}
\max & \sum_{i \in I \cup C} \sum_{j \in F} d_{ij} y_{ij} + \sum_{i \in C} \sum_{l \in L} r_l^i z^l_j \\
\text{subject to:} & \sum_{j \in F} y_{ij} \leq s_i, \quad i \in I \cup C, \\
& \sum_{i,j \in F} y_{ij} - \sum_{l \in L} z^l_j \geq 0, \quad j \in C, \\
& z^l_j \leq D^l_j, \quad l \in L, \quad j \in C, \\
& y_{ij}, z^l_j \geq 0.
\end{align*}
\]

The problem above can be interpreted as follows. There is a set \( I \) of production facilities (with warehouses) and a set of customers \( C \). The set \( F_i \) is the set of facilities that can receive shipments from location \( i \in I \cup C \), i.e. the set of \( F \)easible moves from location \( i \). At the first stage, an amount \( x_{ij} \) is transported from production facility \( i \) to a warehouse or customer location \( j \), before the demand realizations at customer locations become known. After the realization of the demands at customer locations are observed, we move an amount \( y_{ij} \) from location \( i \) to customer location \( j \). At each customer location we face different types of demands, indexed by \( l \in L \). \( D^l_j \) is the units of demand of type \( l \) at location \( j \). We serve \( z^l_j \) units of demand of type \( l \) at location \( j \); the excess demand, if any, is lost. The production capacity of facility \( i \in I \) is denoted \( p_i \).

For the first stage costs, we set \( c_{ij} = c_0 + c_1 \delta_{ij} \), where \( \delta_{ij} \) is the Euclidean distance between locations \( i \) and \( j \), and \( c_0 \) can be interpreted as the unit production cost (if any) and \( c_1 \) is the transportation cost applied on “per mile” basis. For the second stage costs, we have:

\[
d_{ij} = \begin{cases} 
    d_i \delta_{ij} & \text{if } i \in I \\
    d_0 + d_1 \delta_{ij} & \text{if } i \in C.
\end{cases}
\]

\( d_0 \) represents the fixed charge for shipping a unit of the product from one customer location to another customer location, and \( d_i \) are the costs of transportation in the second stage. For every demand type \( l \) occurring in location \( i \), we associate a base revenue \( q^l_i \). We set \( r^l_i = r \cdot q^l_i \). This allows us to scale the revenues of the demands in anyway we like.