ORIE 6334 Approximation Algorithms January 30, 2014

Problem Set 1

Due Date: February 13, 2014

1. W&S Exercise 1.4
2. W&S Exercise 1.5
3. W&S Exercise 1.3
4. W&S Exercise 2.1
5. W&S Exercise 2.11
6. Suppose we have a function f is submodular and nonnegative (i.e. $f(S) \geq 0$ for all $S \subseteq V$), but not necessarily monotone. Consider a local search algorithm which given a current solution S either adds an element $v \in V - S$ to S if $f(S \cup \{v\}) > f(S)$ or deletes an element $v \in S$ if $f(S - v) > f(S)$. Let S^* be a locally optimal solution (we won't worry here about getting one in polynomial time). Let O^* be an optimal solution. In the following we will show that either $f(S^*) \geq \frac{1}{3} f(O^*)$ or $f(V - S^*) \geq \frac{1}{3} f(O^*)$, showing that local search is a $\frac{1}{3}$-approximation algorithm if we can implement it in polynomial time.
   (a) Show that for any $T \subseteq S^*$, $f(S^*) \geq f(T)$ and for any $T \supseteq S^*$, $f(S^*) \geq f(T)$.
   (b) Show that $2f(S^*) + f(V - S^*) \geq f(O^*)$, and conclude the desired result.