ORIE 6334 Approximation Algorithms September 17, 2009

Problem Set 2

Due Date: October 1, 2009

1. Suppose an undirected graph G has a Hamiltonian path. Give a polynomial-time algorithm to find a path of length $\Omega(\log n)$.

2. In the *edge-disjoint paths* problem in directed graphs, we are given as input a directed graph $G = (V, A)$ and k source-sink pairs $s_i, t_i \in V$. The goal of the problem is to find edge-disjoint paths so that as many source-sink pairs as possible have a path from s_i to t_i. More formally, let $S \subseteq \{1, \dots, k\}$. We want to find S and paths P_i for all $i \in S$ such that $|S|$ is as large as possible and for any $i, j \in S$, $i \neq j$, P_i and P_j are edge-disjoint ($P_i \cap P_j = \emptyset$).

   Consider the following greedy algorithm for the problem. Let ℓ be the maximum of $\sqrt{m}$ and the diameter of the graph, where $m = |A|$ is the number of input arcs. For i from 1 to k, we check to see if there exists a s_i-t_i path of length at most ℓ in the current graph. If there is such a path P_i, we add i to S and remove the arcs of P_i from the graph.

   Show that this greedy algorithm is an $\Omega(1/\ell)$-approximation algorithm for the edge-disjoint paths problem in directed graphs.

3. W&S Exercise 4.4

4. W&S Exercise 4.6