ORIE 6334 Approximation Algorithms September 3, 2009

Problem Set 1

Due Date: September 17, 2009

Do any four of the following six problems.

1. W&S Exercise 1.4

2. W&S Exercise 1.1

3. W&S Exercise 1.3

4. (a) W&S Exercise 2.10

   (b) Show that there is no $\left(1 - \frac{1}{e} + \epsilon\right)$-approximation algorithm for the maximum coverage problem for constant $\epsilon > 0$ unless each problem in NP has an algorithm running in $O(n^{O(\log \log n)})$ time (Hint: recall Theorem 1.13 from W&S).

5. (a) W&S Exercise 2.9

   (b) Now we consider the case in which the function f is submodular and non-negative (i.e. $f(S) \geq 0$ for all $S \subseteq V$), but not necessarily monotone. Consider a local search algorithm which given a current solution S either adds an element $v \in V - S$ to S if $f(S \cup \{v\}) > f(S)$ or deletes an element $v \in S$ if $f(S - v) > f(S)$. Let S^* be a locally optimal solution (we won't worry here about getting one in polynomial time). Let O^* be an optimal solution. In the following we will show that either $f(S^*) \geq \frac{1}{3} f(O^*)$ or $f(V - S^*) \geq \frac{1}{3} f(O^*)$, showing that local search is a $\frac{1}{3}$-approximation algorithm if we can implement it in polynomial time.

      i. Show that for any $T \subseteq S^*$, $f(S^*) \geq f(T)$ and for any $T \supseteq S^*$, $f(S^*) \geq f(T)$.

      ii. Show that $2f(S^*) + f(V - S^*) \geq f(O^*)$, and conclude the desired result.

6. W&S Exercise 1.6(b)