Lemma The distance levels k for $k < n-1$ are consecutive (i.e. can't have $B(k+1) \neq \emptyset$, $B(k-1) \neq \emptyset$, but $B(k) = \emptyset$).

Lemma $d(t) \leq |X| - 1$.

Pf Initially, $X = \{s\}$, $d(t) = 0$.

Spse true for X and t. At the end of iter, choose t', set $X' = X \cup \{t\}$. t' has min dist label. Nonempty dist. levels are consecutive

$$\Rightarrow d(t') \leq d(t) + 1 \leq (|X| - 1) + 1 = |X'| - 1.$$

$\square$

Lemma If $i \notin X$, then $d(i) \leq n-2$.

Pf Let $i \notin X$ have max dist label.

Dist. levels $d(t), d(t)+1, \ldots, d(i)$ all non-empty.

$n-|X|-1$ not in X and not t.

$$\Rightarrow \quad d(i) \leq d(t) + (n-|X|-1) \leq |X|-1 + (n-|X|-1)$$

$$= n-2.$$

$\blacksquare$

<u>Lemma</u> Throughout alg. ℓ a cut level s.t. ~~$d(t) < \ell \leq$~~
$d(t) < \ell \leq n-1$.

Remember: k a cut level if $\forall\, i \in B(k)$, and all (i,j)
s.t. $u_f(i,j) > 0$, then $d(i) \leq d(j)$.

<u>Obs</u> <u>If</u> $B(k) = \emptyset$, then k trivially a cut level.

<u>Pf</u> $n-1$ is a cut level, since it is empty.

<u>If</u> we set $\ell \leftarrow i$, then we tried to relabel i,
$|B(d(i))| = 1$. <u>If</u> we were relabeling $i \Rightarrow$

$$d(i) \leq d(j) \quad \forall (i,j) : u_f(i,j) > 0$$

$\Rightarrow d(i)$ is a cut level.

Never set ℓ to $d(t)$: only if $|B(d(t))| = 1$, so that t only node in $B(d(\ell))$. But we never relabel t. $\boxtimes$

<u>Lemma</u> # of relabels is $O(n^2)$

<u>Lemma</u> # of saturating pushes is $O(mn)$.

<u>Lemma</u> # of nonsaturating pushes is $O(mn^2)$.

$\Rightarrow$ Can run Hao-Orlin to find min s-cut in $O(mn^2)$ time.

$\Rightarrow$ Can find global min cut in $O(mn^2)$ time.