ORIE 6300 Mathematical Programming I October 3, 2008

Problem Set 5

Due Date: October 10, 2008

1. The indication for unboundedness in the simplex method shows the existence of feasible solutions to the primal problem with objective function values unbounded below. This implies via weak or strong duality that the dual problem is infeasible. Show how to obtain a short certification of the infeasibility of the dual from the quantities already computed.

2. Consider a LP problem in standard form $\min(cx : x \geq 0, Ax = b)$ with $c \geq 0$. Suppose you run the simplex method for a while. The simplex method terminates if for the current basic dual solution y is dual feasible, i.e., $A_j^T y \leq c_j$ (for $i = 1, \ldots, n$), where A_j is the jth column of A. Suppose you don't have enough patience to wait for this, and so you decide to terminate early: as soon as the current dual solution is *almost feasible*, or more precisely, you stop when $A_j^T y \leq (1+\epsilon)c_j$ (for $j = 1, \ldots, n$) for a given error parameter $\epsilon > 0$.

   What can you say about the current basic solution at this time? Prove that the solution x found at this point is close to optimal, in that it satisfies $cx \leq (1+\epsilon)\min(cx : x \geq 0, Ax = b)$.

3. Consider the linear program $\min(cx : x \geq 0, Ax = b)$. Let B denote an optimal basis. Assume that the problem is generic in that each vertex has a unique basis for which it is the corresponding basic solution.

   Assume now that you want to solve a *parametric* problem, i.e., a set of problems of the form $\min((c + \lambda\bar{c})x : x \geq 0, Ax = b)$, for each possible value of $\lambda \geq 0$. The basis B is a solution for the problem when $\lambda = 0$.

   (a) Prove that the set of values of λ for which basis B is optimal forms an interval $[0, a_1]$. Explain how to compute a_1.

   (b) Show that there is a finite set $a_0 = 0 \leq a_1 \leq \ldots \leq a_k$ and corresponding bases B_i for $i = 0, \ldots, k$ such that $B_0 = B$ and B_i (for $i = 0, \ldots, k$) is the optimal basis if and only if $\lambda \in [a_i, a_{i+1}]$, and B_k is optimal if $\lambda \geq a_k$.