ORIE 6300 Mathematical Programming I November 3, 2014

Problem Set 9

Due Date: November 7, 2014

1. (10 points) Consider the following version of the cutting stock problem. There is a demand b_i for every size s_i and a width W for the raw material, just in the version discussed in class. Change the method from class to work with the following version instead: customers have a 10% tolerance in the order, that is, all the solution has to satisfy is a demand some place between $.9b_i$ and $1.1b_i$ for every size s_i and whatever produced will be bought by the customers. Last time we were minimizing the number of width W raws used. Suppose you are given N (the number of raws you have), and instead you want to maximize the amount of demand satisfied, i.e., if your solution produces p_i finals of size s_i, then you must have that $.9b_i \leq p_i \leq 1.1b_i$ and your goal is to maximize $\sum_i p_i$. Explain how to modify the solution discussed in class to solve this problem.

2. (15 points) Recall the maximum multicommodity flow problem given on the previous problem set. In this problem we are given a directed graph G with nodes V and directed arcs A, and k source-sink pairs (s_i, t_i), where $s_i, t_i \in V$ for $i = 1, \ldots, k$. We may send flow only from a source s_i to the corresponding sink t_i. The goal is to send as much flow as possible from the sources s_i to their corresponding sinks t_i. Each arc $a \in A$ has a capacity u_a; we may not send more than u_a total units of flow through arc a.

   On the last problem set, we used a linear programming formulation of the problem in which there is a variable x_P for each s_i-t_i path P. However, this isn't the only possible linear programming formulation of the problem.

   (a) (5 points) Give another linear programming formulation of the problem which uses variables f^i_{uv} to indicate the amount of flow being sent from s_i to t_i using arc $(u, v) \in A$.

   (b) (8 points) If you've set up your linear programming formulation correctly in the part above, you'll notice that it can be solved via a Dantzig-Wolfe decomposition. What are the linking constraints? What are the associated subproblems? What is an extreme point of the subproblem? How can you tell whether the master problem has a negative reduced cost variable?