ORIE 6300 Mathematical Programming I September 12, 2014

Problem Set 3

Due Date: September 19, 2014

1. Recall the maximum flow problem, and its dual as they were presented in class. We used the variables z^*_{uv} of an optimal dual solution to define $cost(s, v)$ for each vertex v, and then defined the sets $S_\rho = \{v : cost(s, v) \le \rho\}$, and showed that each S_ρ is an s-t cut for $0 \le \rho < 1$. We have also shown that, in any optimal dual solution z^*, at least one of the cuts S_ρ for some $0 \le \rho < 1$ defines a minimum cut by showing that the expected value $E[n(S_\rho)] \le \sum_{(u,v)\in A} z^*_{uv}$.

   Show that all cuts S_ρ occurring with positive probability in the expectation must be minimum s-t cuts for the graph.

2. (Carathéodory's theorem) Show that if $x \in \Re^n$ is a convex combination of $v_1, \ldots, v_k$, then it is also a convex combination of at most $n + 1$ of these points.

3. Consider the polytope $Q = conv(v_1, \ldots, v_k)$ with $v_i \in \Re^n$ for all i. Prove that for any objective function $c \in \Re^n$, there is some v_j such that $c^T v_j \le c^T x$ for all $x \in Q$.