Note: Students are expected to work on homework entirely independently. You may not help other students with homework or seek help on homework from other students.

1. Assume that a stock price follows the Black-Scholes model with \( r = 0.04 \) and \( \sigma = 0.2 \) when time is measured in years. Currently the stock price is $113. A European call option has a strike price equal to $115 and the expiration date is 1/2 year from now. Throughout this problem, assume that \( r \) and \( \sigma \) do not change, as is assumed by the Black-Scholes model. 

[It is recommended that you write a subroutine that calculates the Black-Scholes formula for the price of a call option with \( \sigma, r, K, T, t, \) and \( S_0 \) as input parameters. You will need this subroutine in part (c) and will find it useful for parts (a) and (b).]

(a) What is the price of the option right now (\( t = 0 \))? 
(b) What will be the price of the option 1/4 year from now (when \( t = 1/4 \)) if the stock price is then $115? 
(c) Assume that for all \( t_2 > t_1 \), \( \log(S_{t_2}/S_{t_1}) \) is \( \mathcal{N}\{\mu(t_2-t_1), \sigma^2(t_2-t_1)\} \) where \( \mu = 0.08 \). \footnote{The value of \( \mu \) is not needed for pricing by the Black-Scholes, but is needed to calculate loss distributions. Also, \( \mu \) and \( \sigma \) are related to \( \mu_V \) and \( \sigma^2_V \) on page 332 of the textbook by the equations \( \sigma = \sigma_V \) and \( \mu = \mu_V - \sigma^2/2 \).} Write a program in MATLAB or another language such as R to simulate the distribution of the loss for the next 1/4 year of a portfolio (that is from times 0 to 1/4 year) of a call option on 100 shares. The Monte Carlo sample size should be at least 50,000. 

i. Construct a normal probability plot of the simulation output. How close to normally distribution does the loss distribution appear to be? 
ii. Estimate the mean, standard deviation, skewness, and kurtosis of the loss distribution. \footnote{Skewness and kurtosis of a random variable \( X \) are defined on page 69 of the textbook to be the third and fourth moments of \( \{X - E(X)\}/\sigma_X \). Note that some authors defined kurtosis to be the fourth moment minus 3. I will call that quantity the excess kurtosis, meaning the excess over the value at a normal distribution.} 
iii. Estimate VaR\(_{99}\). Estimate the Monte Carlo variability in the estimate of VaR\(_{99}\).

(d) Approximate the loss distribution using a linear approximation to the loss function. Repeat part (c) using this approximate loss distribution. \( \text{(Hint: The approximate loss distribution is normal.)} \)

2. Prove equation (2.27) on page 46 of the textbook.