Problem 1a. > effectiveSize(univ_t.coda)

k   mu    sigma   tau
955.2276 1413.1482 1500.0000  808.5236

sigma mixes best since it has the larger $N_{\text{eff}}$

Problem 1b. tau mixes worst since it has the smallest $N_{\text{eff}}$

Problem 1c. > summary(univ_t.coda)

Iterations = 1002:2000
Thinning interval = 2
Number of chains = 3
Sample size per chain = 500

1. Empirical mean and standard deviation for each variable, plus standard error of the mean:

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>SD</th>
<th>Naive SE</th>
<th>Time-series SE</th>
</tr>
</thead>
<tbody>
<tr>
<td>k</td>
<td>2.489e+01</td>
<td>11.751537</td>
<td>3.034e-01</td>
<td>3.812e-01</td>
</tr>
<tr>
<td>mu</td>
<td>9.384e-03</td>
<td>0.003580</td>
<td>9.244e-05</td>
<td>9.686e-05</td>
</tr>
<tr>
<td>sigma</td>
<td>6.848e-02</td>
<td>0.002733</td>
<td>7.057e-05</td>
<td>7.061e-05</td>
</tr>
<tr>
<td>tau</td>
<td>2.405e+02</td>
<td>24.279612</td>
<td>6.269e-01</td>
<td>8.619e-01</td>
</tr>
</tbody>
</table>

2. Quantiles for each variable:

<table>
<thead>
<tr>
<th></th>
<th>2.5%</th>
<th>25%</th>
<th>50%</th>
<th>75%</th>
<th>97.5%</th>
</tr>
</thead>
<tbody>
<tr>
<td>k</td>
<td>7.653e+00</td>
<td>1.483e+01</td>
<td>2.316e+01</td>
<td>34.21638</td>
<td>47.76387</td>
</tr>
<tr>
<td>mu</td>
<td>2.205e-03</td>
<td>7.006e-03</td>
<td>9.362e-03</td>
<td>0.01169</td>
<td>0.01649</td>
</tr>
<tr>
<td>sigma</td>
<td>6.345e-02</td>
<td>6.663e-02</td>
<td>6.832e-02</td>
<td>0.07027</td>
<td>0.07402</td>
</tr>
<tr>
<td>tau</td>
<td>2.009e+02</td>
<td>2.235e+02</td>
<td>2.373e+02</td>
<td>255.48430</td>
<td>294.81731</td>
</tr>
</tbody>
</table>

Using the 2.5% and 97.5% percentiles, the 95% equal-tails interval is (7.6, 47.8).

The function HPDinterval() produces a high posterior density interval for each of the three chains. These are (8.2, 47.9), (7.5, 45.3), and (7.5, 48.1).

> options(digits=2)
> HPDinterval(univ_t.coda)

[[1]]

lower   upper
k  8.2e+00  47.898
Below the three chains are combined and the resulting posterior interval is $(7.4, 47)$.

```r
> k1 = univ_t.coda[[1]][1]
> k2 = univ_t.coda[[2]][1]
> k3 = univ_t.coda[[3]][1]
> k = c(k1,k2,k3)
> HPDinterval(as.mcmc(k))
  lower   upper
var1  7.4  47
```

Problem 2a. From the ACF plots, it appears that tau mixes worst and k mixes best. These results do not agree with the results from the $N_{eff}$ values. It should be noted that all of the parameters mix rather well, so it is difficult to determine which parameters mix best and worst.
Problem 2b. The following code extracts the degrees of freedom parameter from each chain, combines the three samples, and computes the sample skewness and kurtosis of the combined sample.

```r
> k1 = univ_t.coda[[1]][,1]
> k2 = univ_t.coda[[2]][,1]
> k3 = univ_t.coda[[3]][,1]
> k = c(k1,k2,k3)
> std_k = (k-mean(k)) / sqrt(mean((k-mean(k))^2))
> options(digits=4)
> mean(std_k^3)
[1] 0.3544
> mean(std_k^4)
[1] 5.012
```

Skewness and kurtosis can also be computed using functions in the `moments` package.

```r
> library(moments)
> skewness(k)
[1] 0.3544
> kurtosis(k)
[1] 2.018
```

Problem 3. The degrees of freedom parameter ($k$) has the most skewed posterior density.
Problem 7. The BUGS code is below and is in the file arma11.bug.

```r
model{
  for (i in 2:N)
  {
    w[i] <- y[i] - phi * y[i-1] - theta * w[i-1]
  }
  w[1] ~ dnorm(0, 0.01)
  for (i in 2:N)
  {
    y[i] ~ dnorm(phi * y[i-1] + theta * w[i-1], tau)
  }
  phi ~ dnorm(0, 0.001)
  theta ~ dnorm(0, 0.001)
  tau ~ dgamma(0.01, 0.0001)
  sigma <- 1/sqrt(tau)
}
```

The R program is below.

```r
library(rjags)
set.seed(5640)
N=600
y = arima.sim(n = N, list(ar = .9, ma = -.5), sd = .4)
y = as.numeric(y)
arma11.sim_data=list(y = y, N = N)
```
init arma11 = function() {
  list(phi = rnorm(1, 0, 0.3),
       theta = rnorm(1, -0.5, 0.1), tau = runif(1, 5, 8))
}

arma11 <- jags.model("arma11.bug", data = arma11.sim_data,
  inits = init arma11,
  n.chains = 3, n.adapt = 1000, quiet = FALSE)
nthin = 5
arma11.coda = coda.samples(arma11, c("phi", "theta", "sigma"),
  n.iter = 500 * nthin, thin = nthin)
summary(arma11.coda)
effectiveSize(arma11.coda)
gelman.diag(arma11.coda)
gelman.plot(univ_t.coda)

The output is below.

> summary(arma11.coda)

Iterations = 1005:3500
Thinning interval = 5
Number of chains = 3
Sample size per chain = 500

1. Empirical mean and standard deviation for each variable,
   plus standard error of the mean:

   Mean     SD  Naive SE Time-series SE
   phi  0.9140 0.0240  0.000620  0.000680
   sigma 0.3920 0.0115  0.000297  0.000324
   theta-0.5690 0.0456  0.001178  0.001309

2. Quantiles for each variable:

   2.5%  25%  50%  75%  97.5%
   phi  0.864  0.899  0.915  0.931  0.958
   sigma 0.371  0.384  0.392  0.400  0.416
   theta-0.656 -0.601 -0.571 -0.541 -0.472

> effectiveSize(arma11.coda)
phi  sigma  theta
   1271   1284   1217

> gelman.diag(arma11.coda)
Potential scale reduction factors:

   Point est. Upper C.I.
   phi     1     1.00
   sigma   1     1.01
(a) The chains mix rather well and all effective sample sizes are over 1,200. In comparison, the actual sample size is not much larger, only 1,500, since there are 3 chains each of size 500 after thinning. The Gelman diagnostics are close to 1 indicating good mixing and an adequate burn-in.

(b) The posterior intervals are below. They differ slightly between chains. The posterior interval for $\phi$ is (0.87, 0.96) for all three chains and contains the true value, 0.9. The interval for $\theta$ is ($-0.66$, $-0.48$) for the second and third chain and ($-0.68$, $-0.50$) for the first. All three of the intervals contain the true value, $-0.5$.

```r
> HPDinterval(arma11.coda)
[[1]]
  lower    upper
phi 0.8729 0.9622
sigma 0.3707 0.4147
theta -0.6679 -0.4961
attr(,"Probability")
```
```
[1] 0.95

[[2]]
  lower  upper
  phi   0.8685  0.9579
  sigma 0.3678  0.4128
  theta -0.6588 -0.4768
attr("Probability")
[1] 0.95

[[3]]
  lower  upper
  phi   0.8676  0.9600
  sigma 0.3686  0.4130
  theta -0.6662 -0.4825
attr("Probability")
[1] 0.95
```