Course Description

On a fundamental level, mathematics is concerned with understanding. It wants to answer questions about how things relate to one another, about what kind of similarities and differences they may have. This course gets to the core of those curiosities. In doing so, we will explore the foundations of mathematical reasoning. These topics include the study of collections of objects (set theory); the structure of statements, arguments, and truth (logic); the interplay between shapes, sizes, and locations (geometry); the examination of randomness and the behavior of uncertainty (probability); and the analysis of information and data (statistics). In the combination of these subjects, this course offers students an enriched quantitative perspective in daily life and a deepened knowledge of mathematics.

Details & Logistics

Instructor: Andrew Daw
Classroom TA: Ms. Jhala
Tutorial TA: Mr. Ferrao
Time & Dates: Wednesday 7:50 – 10:50 AM, January 30th through May 8th, 2019
Textbook: Thinking Mathematically, Robert Blitzer

Learning Objectives

The goal for students is strong knowledge of and proficiency in the mathematical reasoning tools that are covered in this course. This means that at the end of the semester students are expected to be able to:

- Understand and discuss the role of mathematics in problem solving and critical thinking
- Define and recognize sets of objects, perform set operations, and draw and interpret Venn diagrams
- Read and write logical statements, combine logical statements into mathematical arguments and identify conclusions, and create and use truth tables
- Perform computations in different number bases and describe the early history of systems of numeration
- Identify points, lines, and shapes and calculate their areas, circumferences, and volumes
Utilize counting principles, permutations, and combinations to calculate probabilities and apply concepts from set theory to determine the probabilities of events. Analyze sample data in terms of its mean and variance, recognize the shape and characteristics of the normal distribution, and use statistics to solve problems.

List of Topics

Along with the associated book chapters from *Thinking Mathematically*, the topics covered in this course are:

I. Problem Solving and Critical Thinking (*Chapter 1*)
   a. Inductive and Deductive Reasoning (*Section 1.1*)
   b. Estimation, Graphs, Mathematical Models (*Section 1.2*)
   c. Problem Solving (*Section 1.3*)

II. Set Theory (*Chapter 2*)
   a. Basic Set Concepts (*Section 2.1*)
   b. Subsets (*Section 2.2*)
   c. Venn Diagrams and Set Operations (*Section 2.3*)
   d. Set Operations and Venn Diagrams with Three Sets (*Section 2.4*)
   e. Survey Problems (*Section 2.5*)

III. Logic (*Chapter 3*)
   a. Statements, Negations, and Quantified Statements (*Section 3.1*)
   b. Compound Statements and Connectives (*Section 3.2*)
   c. Truth Tables for Negation, Conjunction, and Disjunction (*Section 3.3*)
   d. Truth Tables for the Conditional and the Biconditional (*Section 3.4*)
   e. Equivalent Statements & Variations of Conditional Statements (*Section 3.5*)
   f. Negations of Conditional Statements and De Morgan’s Laws (*Section 3.6*)
   g. Arguments and Truth Tables (*Section 3.7*)

IV. Systems of Numeration (*Chapter 4*)
   a. Our Hindu-Arabic System and Early Positional Systems (*Section 4.1*)
   b. Number Bases in Positional Systems (*Section 4.2*)
   c. Computation in Positional Systems (*Section 4.3*)
   d. Looking Back at Early Numeration Systems (*Section 4.4*)

V. Geometry (*Chapter 10*)
   a. Points, Lines, Planes, and Angles (*Section 10.1*)
   b. Triangles (*Section 10.2*)
   c. Polygons, Perimeter, and Tessellations (*Section 10.3*)
   d. Area and Circumference (*Section 10.4*)
   e. Volume and Surface Area (*Section 10.5*)

VI. Probability (*Chapter 11*)
   a. The Fundamental Counting Principle (*Section 11.1*)
   b. Permutations (*Section 11.2*)
   c. Combinations (*Section 11.3*)
   d. Fundamentals of Probability (*Section 11.4*)
e. Probability with the Fundamental Counting Principle, Permutations, and Combinations (Section 11.5)
f. Events Involving Not and Or; Odds (Section 11.6)
g. Events Involving And; Conditional Probability (Section 11.7)
h. Expected Value (Section 11.8)

VII. Statistics (Chapter 12)
   a. Sampling, Frequency Distributions, and Graphs (Section 12.1)
   b. Measures of Central Tendency (Section 12.2)
   c. Measures of Dispersion (Section 12.3)
   d. The Normal Distribution (Section 12.4)
   e. Problem Solving with the Normal Distributions (Section 12.5)

These may vary depending on the timing of the course. Furthermore, additional topics may be covered if time permits. These may be selected on student preferences. There is a wealth of material in the textbook. For those interested in learning more, the remaining chapters in the book present great opportunities for outside learning. In particular, Chapter 14 may be added if possible.

Weekly Organization

This class will feature a method of instruction that is based around participation and group work. This approach has been shown to be particularly conducive for learning and comprehension. Because this arrangement is somewhat unusual or non-traditional, a recommended weekly schedule is given below. This also details the three segments of each lecture.

<table>
<thead>
<tr>
<th>Wednesday Class (Part 1)</th>
<th>Wednesday Class (Part 2)</th>
<th>Wednesday Class (Part 3)</th>
<th>Thursday through Sunday</th>
<th>Monday &amp; Tuesday</th>
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<tbody>
<tr>
<td>Question and answer session on the prior week’s homework material</td>
<td>New classwork on the concepts from the last week</td>
<td>New material is introduced, and new homework is assigned</td>
<td>Read about the new material, work on homework problems, and develop questions to ask in class and in the math tutorial</td>
<td>Attend the math tutorial on Monday to ask questions to the TA, and revisit the problems on Tuesday ahead of Wednesday’s lecture</td>
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The main exception to this schedule will be when there are exams, which will happen during class time. It is worth noting that most of your learning will come while working on homework problems in the time between classes.

Evaluation & Grading
The assignments and evaluations in this course will be structured and weighted as follows:

- **Midterms (two; 25% each or 20% first and 30% second):** the two midterm evaluations in this course will cover the first and second thirds of the material, respectively.
- **Final (one; 30%):** the final exam will be cumulative, covering all the topics seen in the course.
- **Classwork & Participation (weekly; 15%):** the questions, discussions, and problem solving sessions in class will factor into the grading, with the in-class assignments possibly being collected.
- **Homework (weekly; 5%):** the homework in this course will be graded only for completion and answers will be provided. Therefore, the homework assignments are purely a tool for your learning. It is meant to be used as practice with the concepts. In fact, studies have also shown that working in groups can boost overall learning, so feel free to work together. However, it is strongly recommended that you put serious thought into each question on your own, as the resolution of a problem is how you learn. Documented attendance at the tutorial session is sufficient to receive full credit on homework – if you attend the tutorial, you do not need to turn in homework.

**Semester Outline**

The following table gives a guide of how the semester will go. However, it is important to note that this is a tentative plan and that the weekly topics may very well change. Any major modifications will be announced in class.

<table>
<thead>
<tr>
<th>Week</th>
<th>Date</th>
<th>Topic</th>
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<tbody>
<tr>
<td>1</td>
<td>1/30/2019</td>
<td>Course Introduction</td>
</tr>
<tr>
<td>2</td>
<td>2/6/2019</td>
<td>Set Theory I</td>
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<tr>
<td>3</td>
<td>2/13/2019</td>
<td>Set Theory II</td>
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<tr>
<td>4</td>
<td>2/20/2019</td>
<td>Logic I</td>
</tr>
<tr>
<td>5</td>
<td>2/27/2019</td>
<td>Logic II</td>
</tr>
<tr>
<td>6</td>
<td>3/6/2019</td>
<td>First Midterm</td>
</tr>
<tr>
<td>7</td>
<td>3/13/2019</td>
<td>Numeration</td>
</tr>
<tr>
<td>8</td>
<td>3/20/2019</td>
<td>Geometry I</td>
</tr>
<tr>
<td>9</td>
<td>3/27/2019</td>
<td>Geometry II</td>
</tr>
<tr>
<td>10</td>
<td>4/3/2019</td>
<td>Second Midterm</td>
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<tr>
<td>11</td>
<td>4/10/2019</td>
<td>Probability I</td>
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<tr>
<td>12</td>
<td>4/17/2019</td>
<td>Probability II</td>
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<tr>
<td>13</td>
<td>4/24/2019</td>
<td>Statistics I</td>
</tr>
<tr>
<td>14</td>
<td>5/1/2019</td>
<td>Statistics II</td>
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<tr>
<td>15</td>
<td>5/8/2019</td>
<td>Final</td>
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