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STRUCTURAL VERSUS REDUCED FORM MODELS: A NEW INFORMATION BASED PERSPECTIVE

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This paper compares structural versus reduced form credit risk models from an information based perspective. We show that the difference between these two model types can be characterized in terms of the information assumed known by the modeler. Structural models assume that the modeler has the same information set as the firm's manager—complete knowledge of all the firm's assets and liabilities. In most situations, this knowledge leads to a predictable default time. In contrast, reduced form models assume that the modeler has the same information set as the market—incomplete knowledge of the firm's condition. In most cases, this imperfect knowledge leads to an inaccessible default time. As such, we argue that the key distinction between structural and reduced form models is not whether the default time is predictable or inaccessible, but whether the information set is observed by the market or not. Consequently, for pricing and hedging, reduced form models are the preferred methodology.

1 Introduction

For modeling credit risk, two classes of models exist: structural and reduced form. Structural models originated with Black and Scholes (1973), Merton (1974) and reduced form models originated with Jarrow and Turnbull (1992), and subsequently studied by Jarrow and Turnbull (1995), Duffie and Singleton (1999) among others. These models are viewed as competing (see Bielecki and Rutkowski, 2002; Rogers, 1999; Lando, 2003; Duffie, 2003), and there is a heated debate in the professional and academic literature as to which class of models is best (see Jarrow *et al.*, 2003, and references therein). This debate usually revolves around default prediction and/or hedging performance.

The purpose of this paper is to show that these models are not disconnected and disjoint model types as is commonly supposed, but rather that they are

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really the same model containing different informational assumptions. Structural models assume complete knowledge of a very detailed information set, akin to that held by the firm's managers. In most cases, this informational assumption implies that a firm's default time is predictable. But, this is not necessarily the case.[1] In contrast, reduced form models assume knowledge of a less detailed information set, akin to that observed by the market. In most cases, this informational assumption implies that the firm's default time is inaccessible. Given this insight, one sees that the key distinction between structural and reduced form models is not in the characteristic of the default time (predictable versus inaccessible), but in the information set available to the modeler. Indeed, structural models can be transformed into reduced form models as the information set changes and becomes less refined, from that observable by the firm's management to that which is observed by the market.

An immediate consequence of this observation is that the current debate in the credit risk literature about these two model types is misdirected. Rather than debating which model type is best in terms of forecasting performance, the debate should be focused on whether the model should be based on the information set observed by the market or not. For pricing and hedging credit risk, we believe that the information set observed by the market is the relevant one. This is the information set used by the market, in equilibrium, to determine prices. Given this belief, a reduced form model should be employed. As a corollary of this information structure, the characteristic of the firm's default time is determined—whether it is a predictable or totally inaccessible stopping time.

Surprisingly, at this stage in the credit risk literature, there appears to be *no* disagreement that the asset value process is unobservable by the market (see especially: Duan, 1994; Ericsson and Reneby, 2002, 2003 in this regard). Although not well understood in terms of its implication, this consensus supports the usage of reduced form models. In addition, without the continuously observed firm's asset value, the available information set often implies that the firm's default time is inaccessible, and that a hazard rate model should be applied.

An outline of this paper is as follows. Section 2 sets up the common terminology and notation. Section 3 reviews structural models, and Section 4 reviews reduced form models. Section 5 links the two model types in a common mathematical structure. Section 6 provides the market information based perspective, and Section 7 concludes the paper.

2 The setup

Credit risk investigates an entity (corporation, bank, individual) that borrows funds, promises to return these funds under a prespecified contractual agreement, and who may default before the funds (in their entirety) are repaid.

Let us consider a continuous time model with time period $[0, \bar{T}]$. Given on this time interval is a filtered probability space $\{(\Omega, \mathcal{G}, P), (\mathcal{G}_t : t \in [0, \bar{T}])\}$ satisfying the usual conditions, with P the statistical probability measure. The information set $(\mathcal{G}_t : t \in [0, \bar{T}])$ will be key to our subsequent arguments. We take the perspective of a modeler evaluating the credit risk of a firm. The information set is that information held by the modeler. Often, in the subsequent analysis, we will be given a stochastic process Yt, that may be a vector valued process, and we will want to study the information set generated by its evolution. We denote this information set by $\sigma(Y_s : s \leq t)$.

Let us start with a generic firm, that borrows funds in the form of a zero-coupon bond promising to pay a dollar (the face value) at its maturity, time

$T \in [0, \bar{T}]$. Its price at time $t \leq T$ is denoted by $v(t, T)$. Let this be the only liability of the firm. Also traded are default free zero-coupon bonds of all maturities, with the default free spot rate of interest denoted by r_t. Markets for the firm's bond and the default free bonds are assumed to be arbitrage free, hence, there exists an equivalent probability measure Q such that all discounted bond prices are martingales with respect to the information set $\{\mathcal{G}_t : t \in [0, \bar{T}]\}$. The discount factor is $e^{-\int_0^t r_s ds}$. Markets need not be complete, so that the probability Q may not be unique.

3 Structural models

This section reviews the structural models in an abstract setting. As mentioned in the introduction, structural models were introduced by Black and Scholes (1993) and Merton (1974). The simplest structural model is used to illustrate this approach. The key postulate emphasized in our presentation is that the information set $\{\mathcal{G}_t : t \in [0, \bar{T}]\}$ that the modeler observes contains the filtration generated by the firm's asset value. Let the firm's asset value be denoted by A_t. Then, $\mathcal{F}_t = \sigma(A_s : s \leq t) \subset \mathcal{G}_t$.

Let the firm's asset value follow a diffusion process that remains non-negative:

$$dA_t = A_t\alpha(t, A_t)dt + A_t\sigma(t, A_t)dW_t \quad (1)$$

where α_t, σ_t are suitably chosen so that expression (1) is well defined, and Wt is a standard Brownian motion. See, for example, Theorem 71 on page 345 of Protter (2004), where such equations are treated.

Given the liability structure of the firm, a single zero-coupon bond with maturity T and face value 1, default can only happen at time T. And, default happens only if $A_T \leq 1$. Thus, the probability of default on this firm at time T is given by

$$P(A_T \leq 1).$$

The time 0 value of the firm's debt is

$$v(0, T) = E^Q\left([\min(A_T, 1)]e^{-\int_0^T r_s ds}\right). \quad (2)$$

Assuming that interest rates r_t are constant, and that the diffusion coefficient $\sigma(t, \omega, x) = \sigma$ is constant (that is, the firm's asset value's volatility is constant), this expression can be evaluated in closed form. The expression is

$$v(0, T) = e^{-rT}N(d_2) + A_tN(-d_1) \quad (3)$$

where $N(\cdot)$ is the cumulative standard normal distribution function, $d1 = [\log(At) + (r + \sigma^2/2)T]/\sigma\sqrt{T}$, and $d_2 = d_1\sigma\sqrt{T}$.

This is the original risky debt model of Black and Scholes (1973) and Merton (1974), where the firm's equity is viewed as a European call option on the firm's assets with maturity T and a strike price equal to the face value of the debt. To see this, note that the time T value of the firm's equity is: $A_T - \min(A_T, 1) = \max(A_T - 1, 0)$. The right-hand side of this expression corresponds to the payoff of the previously mentioned European call option on the firm's time T asset value.

Because the Black–Scholes and Merton model has default only occurring on one date, the model has since been generalized to allow default prior to time T if the asset's value hits some prespecified default barrier, L_t. The economic interpretation is that the default barrier represents some debt covenant violation. In this formulation, the barrier itself could be a stochastic process. Then, the information set must be augmented to include this process as well, i.e. $\mathcal{F}t = \sigma(A_s, L_s : s \leq t)$. We assume that in the event of default, the debt holders receive the value of the barrier at time T. Other formulations are possible.[2] In this generalization, the default time becomes a random variable and it corresponds to the first hitting time of the barrier:

$$\tau = \inf\{t > 0 : A_t \leq L_t\}. \quad (4)$$

Here, the default time is a predictable stopping time.[3]

Formally, a stopping time τ is a non-negative random variable such that the event $\{\tau \leq t\} \in \mathcal{F}t$ for every $t \in [0, \bar{T}]$ (see Protter, 2004). A stopping time τ is predictable if there exists a sequence of stopping times $(\tau_n)_{n\geq 1}$ such that τ_n is increasing, $\tau_n \leq \tau$ on $\{\tau > 0\}$ for all n, and $\lim_{n\to\infty}\tau_n = \tau$ a.s. Intuitively, a predictable stopping time is "known" to occur "just before" it happens, since it is "announced" by an increasing sequence of stopping times. This is certainly the situation for the structural model constructed above. In essence, although default is an uncertain event and thus technically a surprise, it is not a "true surprise" to the modeler, since it can be anticipated with almost certainty by watching the path of the asset's value process. The key characteristic of a structural model that we emphasize in this paper is the observability of the information set $\mathcal{F}_t = \sigma(A_s, L_s : s \leq t)$, and not the fact that the default time is predictable.

Given the default time in expression (4), the value of the firm's debt is given by

$$v(0,T) = E\left([1_{\{\tau\leq T\}}L_\tau + 1_{\{\tau>T\}}1]\mathrm{e}^{-\int_0^T r_s \mathrm{d}s}\right). \tag{5}$$

If interest rates r_t are constant, the barrier is a constant L, and the asset's volatility σ_t is constant; then, expression (5) can be evaluated explicitly, and its value is

$$v(0,T) = L\mathrm{e}^{-rT}Q(\tau \leq T) + \mathrm{e}^{-rT}[1 - Q(\tau \leq T)], \tag{6}$$

where $Q(\tau \leq T) = N(h_1(T)) + A_0\mathrm{e}^{(1-2r/\sigma^2)} \times N(h_2(T))$, $h_1(T) = [-\log A_0 - (r - \sigma^2/2)T]/\sigma\sqrt{T}$, and $h_2(T) = [-\log A_0 + (r - \sigma^2/2)T]/\sigma\sqrt{T}$ (see Bielecki and Rutkowski, 2002, p. 67).

Returning again to the more general risky debt model contained in expression (5), note that interest rates are stochastic. Using the simple zero-coupon bond liability structure as expressed above, this more general formulation includes the models of Shimko *et al.* (1993), Nielsen *et al.* (1993), Longstaff and Schwartz (1995), and Hui *et al.* (2003). Generalizations of this formulation to more complex liability structures include the papers by Black and Cox (1976), Jones *et al.* (1984), and Tauren (1999).

From the perspective of this paper, the key assumption of the structural approach is that the modeler has the information set generated by continuous observations of both the firm's asset value and the default barrier. This is equivalent to the statement that the modeler has continuous and detailed information about all of the firm's assets and liabilities. This is the same information set that is held by the firm's managers (and regulators in the case of commercial banks). This information set often implies that the default time is predictable, but this is not necessarily the case.

4 Reduced form models

This section reviews reduced form models in an abstract setting. Reduced form models were originally introduced by Jarrow and Turnbull (1992) and subsequently studied by Jarrow and Turnbull (1995), and Duffie and Singleton (1999), among others. As before, the simplest structure is utilized to illustrate the approach. The key postulate emphasized here is that the modeler observes the filtration generated by the default time τ and a vector of state variables X_t, where the default time is a stopping time generated by a Cox process $N_t = 1_{\{r\leq t\}}$ with an intensity process λ_t depending on the vector of state variables X_t (often assumed to follow a diffusion process), i.e. $\mathcal{F}_t = \sigma(\tau, X_s : s \leq t) \subset \mathcal{G}_t$. Intuitively, a Cox process is a point process where conditional on the information set generated by the state variables over the entire time interval

$\sigma(X_s : s \leq \bar{T})$, the conditioned process is Poisson with intensity $\lambda_t(X_t)$, see Lando (1998). In reduced form models, the processes are normally specified under the martingale measure Q.

In this formulation, the stopping time is totally inaccessible. Formally, a stopping time τ is a totally inaccessible stopping time if, for every predictable stopping time S, $Q\{\omega : \tau(\omega) = S(\omega) < \infty\} = 0$.[4] Intuitively, a totally inaccessible stopping time is not predictable, that is, it is a "true surprise" to the modeler. The difference between predictable and totally inaccessible stopping times is not the key distinction between structural and reduced form models that we emphasize herein.

To complete this formulation, we also give the payoff to the firm's debt in the event of default, called the recovery rate. This is usually given by a stochastic process δ_t, also assumed to be part of the information set available to the modeler, i.e. $\mathcal{F}_t := \sigma(\tau, X_s, \delta_s : s \leq t)$. This recovery rate process can take many forms [see Bakshi *et al.* (2001) in this regard]. We assume, to be consistent with the structural model in the previous section, that the recovery rate δ_τ is paid at time T.

We emphasize that the reduced form information set requires less detailed knowledge on the part of the modeler about the firm's assets and liabilities than does the structural approach. In fact, reduced form models were originally constructed to be consistent with the information that is available to the market.

In the formulation of the reduced form model presented, the probability of default prior to time T is given by

$$\begin{aligned} Q(\tau \leq T) &= E^Q\left(E^Q(N(T) = 1 \mid \sigma(X_s : s \leq \bar{T}))\right) \\ &= E^Q\left(e^{-\int_0^T \lambda_s ds}\right). \end{aligned} \tag{7}$$

The value of the firm's debt is given by

$$\begin{aligned} v(0, T) &= E\left(\left[1_{\{\tau \leq T\}}\delta_\tau + 1_{\{\tau > T\}}1\right] e^{-\int_0^T r_s ds}\right). \end{aligned} \tag{8}$$

Note that a small, but crucial distinction between the pricing expression (5) in the structural model and the pricing expression (8) in the reduced form model is that the recovery rate process is prespecified by a knowledge of the liability structure in the structural approach, while here it is exogenously supplied. This distinction is characteristic of a reduced form model to the extent that the liability structure of the firm is usually not continuously observable, whereas the resulting recovery rate process is.

This formulation includes Jarrow and Turnbull (1995), Jarrow *et al.* (1997), Lando (1998), Duffie and Singleton (1999), Madan and Unal (1998), among others. For example, if the recovery rate and intensity processes are constants (δ, λ), then this expression can be evaluated explicitly, generating the model in Jarrow and Turnbull (1995) where the debt's value is given by

$$v(0, T) = p(0, T)\left(\delta + (1 - \delta)e^{-\lambda T}\right) \tag{9}$$

where $p(0, T) = E^Q\left(e^{-\int_0^T r_s ds}\right)$. Other extensions of this model include the inclusion of counterparty risk, see Jarrow and Yu (2001).

5 A mathematical overview

This section relates structural models to reduced form models by concentrating on the information sets held by the modeler. We claim that if one changes the information set held by the modeler from more to less information, then a structural model with default being a predictable stopping time can be transformed into a hazard rate model with default being an inaccessible stopping time.

To see this, we imbed the different approaches into one unifying mathematical framework.

Let us begin with a filtered complete probability space $(\Omega, \mathcal{G}, P, \mathbb{G})$, where $\mathbb{G} = (\mathcal{G}_t)_{t\geq 0}$. The information set $\mathbb{G}$ is the information set available to the modeler. On this space, we assume a given Markov process $A = (A_t)_{t\geq 0}$, where A_t represents the firm's value at time t. Often one assumes that A is a diffusion and that it solves a stochastic differential equation driven by Brownian motion and the Lebesgue measure. For simplicity, let us assume exactly that; i.e.

$$A_t = 1 + \int_0^t \sigma(s, A_s)\mathrm{d}B_s + \int_0^t \mu(s, A_s)\mathrm{d}s.$$

In a structural model, one assumes that the modeler observes the filtration generated by the firm's asset value $\mathbb{F} = (\sigma(A_s : 0 \leq s \leq t))_{t\geq 0}$ from which one can divine the coefficients σ and μ. Consequently, $\mathbb{F} \subset \mathbb{G}$.

Default occurs when the value of the firm crosses below some threshold level process $L = (L_t)_{t\geq 0}$, where it is assumed that once default occurs, the firm cannot recover. While this default barrier can be a stochastic process, for the moment we assume that it is a constant. We then let

$$\tau = \inf\{t > 0 : X_t \leq L\}.$$

Default occurs at the time τ. The stopping time τ will then be $\mathbb{G}$ predictable.

As an alternative to the simple model just described, it is reasonable to assume that the modeler does not continuously observe the firm's asset value. Indeed, there appears to be *no* disagreement in the literature that the asset value process is unobservable [see especially Duan (1994) and Ericsson and Reneby (2002, 2003) in this regard]. Then, given the modeler has partial information, the question then becomes how to model this partial information?

A method proposed by Duffie and Lando (2001) is to obscure the process A by only observing it at discrete time intervals (not continuously) and by adding independent noise. One then obtains a discrete time process $Z_t = A_t + Y_t$, where Y_t is the added noise process, and which is observed at times t_i for $i = 1, \ldots, \infty$. Since one sees Z and not A, one has a different filtration of observable events: $\mathbb{H} = (\mathcal{H}_t)_{t\geq 0} \subset \mathbb{G}$, where $\mathcal{H}_t = \sigma(Z_{t_i} : 0 \leq t_i \leq t)$, up to null sets. Here, τ need not be a stopping time for the filtration $\mathbb{H}$. One is now interested in the quantity

$$P(\tau > u \mid \mathcal{H}_t).$$

However, for this example, Duffie and Lando assume a structure such that they are able to show that τ is still a stopping time, but one that is totally inaccessible with $1_{\{t\geq\tau\}} - \int_0^t \lambda(s, \omega)\mathrm{d}s$ a local martingale. Colloquially, one says that the stopping time τ has the intensity process $\lambda = (\lambda_t)_{t\geq 0}$.[5]

The reason for this transformation of the default time τ from a predictable stopping time to an inaccessible stopping time is that between the time observations of the asset value, we do not know how the asset value has evolved. Consequently, prior to our next observation, default could occur unexpectedly (as a complete surprise). Under this coarser information set, the price of the bond is given by expression (8) with the recovery rate L. Here, the structural model, due to both information obscuring and reduction, is transformed into an intensity based hazard rate model.

Kusuoka (1999) has proposed a more abstract version of the Duffie–Lando model where the relevant processes are observed continuously and not discretely. In this approach, Kusuoka does not specify where the default time τ comes from, but rather he begins with the modeler's filtration $\mathbb{H} \subset \mathbb{G}$ and a positive random variable τ. He then expands the filtration "progressively" so that in the expanded filtration $\mathbb{J} \subset \mathbb{G}$ the random variable τ is a stopping time. (The interested reader can consult Chapter VI of Protter (2004) for an introduction to the theory of the expansion of filtrations.) This filtration expansion is analogous to adding noise to the asset value process as in Duffie and Lando. Unfortunately, Kusuoka makes the rather restrictive hypothesis that all H martingales remain martingales in the larger filtration $\mathbb{J}$, thereby limiting the application of his model.

Another variant of introducing noise to the system is that proposed by Giesecke and Goldberg (2003), where the default barrier itself is taken to be a random curve. (This is usually taken to be a horizontal line of the form $y = L$, but where the level L itself is unknown, and modelled by making it random.) The modeler cannot see this random curve, which is constructed to be independent of the underlying structural model. Since the default time depends on a curve that cannot be observed by the modeler, the default time τ is rendered totally inaccessible. Nonetheless, Giesecke and Goldberg still assume that the firm's asset value process is observed continuously, making their structure a kind of hybrid model, including both reduced form elements and strong assumptions related to structural models, namely that one can observe the structural model.

Çetin *et al.* (2004) take an alternative approach to those papers just cited. Instead of adding noise to obscure information as in Duffie–Lando, or beginning with the investor's filtration and expanding it as in Kusuoka, Çetin *et al.* begin with a structural model as in Duffie–Lando, but where the modeler's filtration $\mathbb{G}$ is taken to be a strict subfiltration of that available to the firm's managers. Starting with the structural model similar to that described above, Çetin *et al.* redefine the asset value to be the firm's cash flows. The relevant barrier is now $L_t = 0$, all $t \geq 0$. Here, the modeler only observes whether the cash flows are positive, zero, or negative. They assume that the default time is the first time, after the cash flows are below zero, when the cash flow both remains below zero for a certain length of time and then doubles in absolute magnitude. Under this circumstance, the default time is totally inaccessible and the point process has an intensity, yielding an intensity based hazard rate model.

The approach of Çetin *et al.* involves what can be seen as the obverse of filtration expansion: namely filtration shrinkage. However, the two theories are almost the same: after one shrinks the filtration, in principle, one can expand it to recover the original one. Nevertheless, the mathematics is quite different, and it is perhaps more natural for the default time to be a stopping time in both filtrations, and to understand how a predictable default time becomes a totally inaccessible time when one shrinks the filtration.

A common feature of these approaches to credit risk is that for the modeler's filtration $\mathbb{H}$, the default time τ is usually not a stopping time, nor does it change in nature from a predictable stopping time to a totally inaccessible stopping time. One can now consider the increasing process $1_{\{t \geq \tau\}}$, which may or may not (depending on the model) be adapted to $\mathbb{H}$. In any event, a common thread is that $1_{\{t \geq \tau\}}$ is, or at least its projection onto $\mathbb{H}$ is, a submartingale [see, e.g. Protter (2004) for the fact that its projection is a submartingale], and the increasing process Λ of its Doob–Meyer decomposition can be interpreted as its compensator in $\mathbb{H}$. Since τ is totally inaccessible, Λ will be continuous, and usually one can verify that Λ is of the form $\Lambda_t = \int_0^t \lambda_s \mathrm{d}s$, where the process λ then plays the role of its arrival intensity under the filtration $\mathbb{H}$.

Thus, the overall structure is one of filtrations, often with containment relationships, and how stopping times behave in the two filtrations. The structural models play a role in the determination of the structure that begets the default time; but as the information available to the modeler is reduced or obscured, one needs to project onto a smaller filtration, and then the default time (whether changed, obscured, or remaining the same) becomes totally inaccessible, and the compensator Λ of the one jump point process $1_{\{t \geq \tau\}}$ becomes the object of interest. If Λ can be written in the form $\Lambda_t = \int_0^t \lambda_s \mathrm{d}s$, then the process λ can be interpreted as the instantaneous rate of default, given the modeler's information set.

6 Observable information sets

In the mathematical overview, we observed that a structural model's default time is modified when one changes the information set used by the modeler. Indeed, it can be transformed from a predictable stopping time to a totally inaccessible stopping time. From our perspective, the difference between structural and reduced form models does not depend on whether the default time is predictable or inaccessible, but rather on whether the default time is based on a filtration that is observed by the market or not.

Structural models assume that the modeler's information set $\mathbb{G}$ is that observed by the firm's managers—usually continuous observations of the firm's asset value A_t and liabilities L_t processes. In contrast, reduced form models assume that the modeler's information set $\mathbb{G}$ is that observed by the market, usually the filtration generated by a stopping time τ and continuous observations of a set of state variables X_t. As shown in the previous section, one can start with the larger information set $\mathbb{G}$, modify it to a reduced information set $\mathbb{H}$, and transform a structural model to a reduced form model. In the review of the literature contained in the previous section, using this classification scheme, we see that the model of Giesecke and Goldberg (2003) is still a structural model, even though it has a totally inaccessible stopping time, while the models of Duffie and Lando (2001), Kusuoka (1999), and Çetin *et al.* (2004) are reduced form models. Giesecke and Goldberg's (2003) is still a structural model because it requires the continuous observation of the firm's asset value, which is not available to the market, while in the models of Duffie and Lando (2001), Kusuoka (1999), and Çetin *et al.* (2004), the information sets assumed are those available to the market.

Which model is preferred—structural or reduced form—depends on the purpose for which the model is being used. If one is using the model for risk management purposes—pricing and hedging—then the reduced form perspective is the correct one to take. Prices are determined by the market, and the market equilibrates based on the information that it has available to make its decisions. In marking-to-market, or judging market risk, reduced form models are the preferred modeling methodology. Instead, if one represents the management within a firm, judging its own firm's default risk for capital considerations, then a structural model may be preferred. However, this is not the approach one wants to take for pricing a firm's risky debt or related credit derivatives.

7 Conclusion

The informational perspective of our paper implies that to distinguish which credit risk model is applicable, structural or reduced form, one needs to understand what information set is available to the modeler. Structural models assume that the information available is that held by the firm's managers, while reduced form models assume that it is the information observable to the market. Given this perspective, the defining characteristics of these models is not the property of the default time—predictable or inaccessible—but rather the information structure of the model itself.

If one is interested in pricing a firm's risky debt or related credit derivatives, then reduced form models are the preferred approach. Indeed, there is consensus in the credit risk literature that the market does *not* observe the firm's asset value continuously in time. This implies, then, that the simple form of structural models illustrated in Section 3 above does not apply. In contrast, reduced form models have been constructed, purposefully, to be based on the information available to the market.

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Notes

1 An example would be where the firm's asset value follows a continuous time jump diffusion process.

2 For example, this could be easily modified to be the value of the barrier paid at the default time τ. This simple recovery rate process is selected to simplify expression (6) below.

3 If the asset price admits a jump, then the default time usually would not be predictable. From the perspective of this paper, however, this would still be called a structural model.

4 Recall that the formulation of most reduced form models takes place under the martingale probability measure Q.

5 It is not always the case that the compensator of a totally inaccessible stopping time has an intensity. Having an intensity is equivalent to requiring that the compensator has absolutely continuous paths. When this happens is an open question, and a subject of current study. See pages 191–193 of Protter (2004), and also Zeng (2004) for preliminary results on this topic.

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