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Joseph Leo Doob, 1910–2004

Joseph Leo Doob died June 7, 2004, in Urbana, Illinois, where he had lived most of his life. Known to many as Joe Doob, or just Joe, he and his work had a profound influence on 20th century mathematics, science, and technology. This influence will long continue.

Doob filled many roles during his life: devoted husband and father, and mentor to many a young mathematician, and a world leader in the then infant area of modern probability theory. It is hard to appreciate his impact on the field of probability in the United States, which at the time was not considered by most mathematicians to be a branch of mathematics. Doob contributed to many mathematical fields always with an emphasis on rigor. In the early part of the last century, this was unusual in the work of many probabilists. Doob played a large role in changing the attitudes towards Probability Theory of mathematicians across the United States, by illustrating how one could rigorously prove new theorems in potential theory using Brownian motion and probabilistic techniques in the then highly popular research area of potential theory. He was a no nonsense, generous person, and a visionary in terms of his early and sustained opposition to antisemitism (which was natural for him), racism and sexism.

Other obituaries describing the role of Doob in mathematics have been published or will be soon. (See for example [1]. Also the *Annals of Probability* will have articles remembering Doob.) Instead of adding to the numbers, we prefer to remember Doob in these pages from his social side, integrated as it was within his academic life and friends. On October 10 of 2004, a celebration of his life and work was held in Urbana, where longstanding friends and colleagues of Joe Doob were asked to comment on him, and their remarks revolved around one of his great loves, the Saturday Hike. We now publish these remarks, which have been slightly edited, preceded by a quick resume of some highlights of his mathematical career.

Remarks on the impact of Doob's career

Doob is remembered as the person who developed martingale theory, which at the beginning he did almost single handedly. He worked in martingale theory from

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around 1940 to 1980. His work laid the basis for Shizuo Kakutani's ground breaking paper in 1944 showing the connection between martingales and harmonic functions. In 1945 Doob clearly and carefully states the strong Markov property for the first time, showing it holds in some special cases. In the 1940s, in their writings Doob and K. Itô rewrote much of P. Lévy's work using a rigorous measure theoretic basis.

In 1953 Doob published his book *Stochastic Processes* [3]. This book can be seen as a turning point, introducing probability theory into the modern mathematical spirit of the day. Paul André Meyer has described it as "the Bible of the new probability theory" and remarking further that "the special status of Doob (besides his own numerous results) comes from his familiarity with measure theory, which he embraced as fundamental to probability, without any loss of intuition."[1] Doob was prescient in his choice of topics in that book: σ algebra filtrations, the concept of adapted processes, stopping times, martingale theory in both discrete and continuous time, Markov chains, and the Itô calculus and diffusions. These form a core of today's interests among researchers. But even more important, Doob's book established probability theory as a respectable subject within mathematics. Probability had previously suffered under a cloud of suspicion among mathematicians, who were not sure what the subject really was (was it statistics? a special use of measure theory? physics?). Doob's book, and his own work, especially showing the connections between martingale theory and axiomatic potential theory, did much to dispel that.

As well as the many research articles Doob wrote and published, he wrote two additional books. In 1984 his spectacular tome *Classical Potential Theory and Its Probabilistic Counterpart* [4] appeared. He had worked on this book of 846 pages for a long time, which is split into two halves: a classical potential theory first half, and a probabilistic potential theory second half. In the second half, one can see how much of modern stochastic processes has probabilistic potential theory as its wellspring. Then in 1994, in a surprise to many, Doob published the beautiful little book *Measure Theory* [5]. P.A. Meyer wrote in a book review of this book [7] "All recent books on measure theory are 'good' books, well planned for students, and they are very much alike. I must say our library has stopped buying them. It will buy this one." He goes on to explain how Doob had managed to be creative and innovative with one of the older, more well-established subjects.

Hopefully, one will gain some insight into Doob's wonderful personality in the remarks which follow. But we cannot resist to quote from the first paragraph of the introduction of [5], which is classic Doob: "This book was planned originally not as work to be published, but as an excuse to buy a computer, incidentally to give me a chance to organize my own ideas on what measure theory every would-be analyst

[1] C. Dellacherie and P.A. Meyer, in one of their volumes [2, p. 184, number IV.49], also comment on his invention of stopping times: "Il a sans doute fallu autant de génie aux créateurs du calcul infinitésimal pour expliciter la notion si simple de drivée, qu'à leurs successeurs pour faire tout le reste. L'invention des temps d'arrêt par Doob est tout à fait comparable." Michel Emery [6] has commented that one could change that quotation to "the triangle: filtrations—stopping times—martingales; it is an astonishing feat of strength because he had to invent all three of them at the same time: any two are not sufficient to build upon."

should learn, and to detail my approach to the subject. When it turned out that Springer-Verlag thought that the point of view in the book had general interest and offered to publish it, I was forced to try to write more clearly and search for errors. The search was productive.''

On a personal note, around 15 years ago Doob told the second author he spent his days searching the web for his name. Calling his bluff, the second author wrote on his own web page, ''Greetings to Joe Doob, who may find this when surfing the web.'' Two years later he received an e-mail from Doob, not mentioning the web page, but stating in concise simplicity ''Have seen greetings, and return same.''

Record of the Celebration of the Life of Joseph Leo Doob (Held in October 2004, in Urbana, Illinois, USA)

One of the important elements in Doob's life was the Saturday Hike. Another important element was music. So it was fitting that the Celebration began with music played by another Saturday Hiker, Rudolph Haken. Professor Haken, a faculty member of the School of Music, played three movements of the viola version of J.S. Bach's Suite III in C-Major, BMV 1009, originally composed for the unaccompanied cello. The speakers followed. Here their remarks about Doob are revisions and expansions of their oral presentations.

Steve Doob (the oldest of Joe and Elsie's three children): First, I'd like to say how devoted my father was to an organization called ''The Hike.'' It is made up mostly of university professors who meet every Saturday afternoon at 2 and drive to one of the innumerable spots along local rivers where they have been hiking for nearly a century. Some of them make a campfire, while the more adventuresome build up their appetites by hiking for an hour or so. And a good appetite is what you need for the meal that follows. From the heart healthy ''Hikers' Delight'' (onions, cheese and hot peppers fried in bacon grease) slathered over rock hard, homemade German bread, to the steaks, cookies and stollen, no one goes home hungry.

I left Urbana in 1959 with fond memories of all the unique individuals I met on the Hike while in high school in the fifties and later during my occasional visits home. Of course, there were many people from the Math Department, but I also remember being impressed by representatives of other departments as well. There were specialists in Literature, Economics, Herpetology, Physics, Physical Education and Geology, to name a few. What I hadn't been exposed to before were the adults who discussed and bantered in a lighthearted and irreverent way about the issues of the day. What made it all the more interesting was the frequent disagreements, which resulted in bets of ''Pie for the Hikers.'' (The loser had to provide a pie the following week.)

Consider this a plug for the Hike from my father. At his request his ashes were scattered by Hikers at one of his favorite spots. Several times in his last few months, he expressed the wish that the Hike would outlive him. So far, it has. When I joined it for the Doob Memorial Hike on October 9, there were around 35 people. They were subdued and a little too reverent, however. My father would have been pleased

at the large crowd but would have abhorred the reverence. From now on, I hope that discussions will be livelier and that the tradition of rashly held opinions that lead to delicious pies will continue. As you might imagine, my father did not believe in ghosts, but just in case he has become one anyway, I'm sure he would want to be suitably entertained when he joins the Hikers around the campfire on Saturday nights.

I'd say that one of the keys to my father's impressive mathematical legacy can be found on the Hike, and I suggest you try going out on it sometime. For information, contact the current "Commissar," Sam Wagstaff, at ssw@cerias.purdue.edu.

As a child, I didn't appreciate my father at all. I had no idea that his taking me fishing nearly every week and his taking his family on summer vacations to lakes every year were something special. I know it must have happened, but I can't remember any time that his work interfered with my childhood. What I value now is the independence that he exemplified and fostered in me. He was not one to follow the crowd. He was skeptical of all strongly held opinions and beliefs, and especially enjoyed puncturing any pomposity he detected. When I didn't agree with him, he relished the argument so much that if I finally gave in he was liable to change sides just to keep the argument going.

He didn't pass on any of his mathematical abilities to me, but he taught me how to fix things that were broken. I learned how to solve electrical, plumbing and carpentry problems as a child, and this led to my house restoration and auto body careers as an adult. He showed me that you can always find a flaw in your work if you look hard enough; no matter how many you find there is always "one more." I learned that no matter how many cucumber beetles you kill in your garden or how many times you pore over your writing, there is always another beetle or another mistake that you've missed. Another early lesson I remember was that misbehaving machinery often responded favorably to a swift kick. (Could this have led to my 40-year martial art practice?)

I find it amazing that my father, who grew up in New York City, could find life so satisfying in Urbana; that he could resist the offers of positions at more prestigious institutions; and that he could do so much in his career while still doing so much with his family. I know he would have been embarrassed by all the attention at this memorial, but at the same time I know he would have been very pleased.

Don Burkholder: Doob's mathematical work will be discussed in a number of journals: the Notices of the American Mathematical Society, the Annals of Probability, the Illinois Journal of Mathematics, and others. Our focus here is primarily on his life. However, let me mention some of the areas of mathematics to which he made important contributions: complex function theory, ergodic theory, martingale theory, mathematical statistics, Markov processes, the general theory of stochastic processes, probabilistic potential theory, and axiomatic potential theory which is nonprobabilistic. For some of these, Doob was the leading pioneer.

In 1987, Paul-Andre Meyer, no longer living now but then a mathematical star in France, especially in probability and potential theory, wrote "Personally, I think he is one of the really great mathematicians of the century."

In addition to his research, Doob served the mathematical profession as president of the Institute of Mathematical Statistics in 1950, as president of the American Mathematical Society during 1963 and 1964, and in many other capacities.

He was elected to the National Academy of Sciences in 1957 and to the American Academy of Arts and Sciences in 1965. He was also made a foreign associate of the French Academy of Sciences in 1975. He was awarded the National Medal of Science at a ceremony in the White House in 1979. Being an informal man, he was not wearing a necktie. In 1984 he was given the Steele Prize by the American Mathematical Society for his outstanding career and for his "continuing profound influence." He received many other honors, including an honorary doctoral degree from the University of Illinois in 1981. He took all of these honors lightly.

David Blackwell (One of Doob's early students, Blackwell received four degrees from the University of Illinois the last one being an honorary degree. He is a member of the National Academy of Sciences and has received much additional recognition for his many contributions to mathematics, including mathematical statistics. He was not able to be present at the Celebration but did send the following message):

"One of the luckiest accidents of my life was having Joe Doob as my thesis adviser. It happened like this. Early in my second year as a graduate student at Illinois, a fellow graduate student, Don Kibbey, asked me who my thesis adviser was. I replied that I didn't have one and should start looking. Then Don said 'Why don't you try Joe Doob? He's my adviser, he's a nice guy, and I'll bet he'll take you.' I'd never met Joe and scarcely knew who he was, but I trusted Don and asked Joe, and he ACCEPTED. At the time I didn't even know that he worked in probability, just that he was an analyst. At that time probability at Illinois was taught by a very nice old professor, Arthur Crathorne."

"For the rest of that year Joe didn't give me a thesis topic, just measure theory and probability papers to read, including Kolmogorov's Grundbegriffe. Later that year Joe gave me his own path-making paper Stochastic processes with an integral-valued parameter to read. That paper was my constant companion for years. It was only after reading it that I realized that I'd been lucky enough to get one of the founders of modern probability as my thesis adviser."

"At the end of that year, Joe gave me two thesis topics, one of which involved his recent discovery that what he called sequences with property E converge backward. Joe knew that these sequences were important, and for several years wondered why they didn't catch on. Finally he renamed them "backward martingales", and then they did catch on quickly. Nowadays every probabilist anywhere knows what a martingale is."

"While I was writing my thesis, Joe and two of his recent students, Paul Halmos and Warren Ambrose, were planning to spend the next year at the Institute for Advanced Study. Joe decided that I needed to spend the next year there also, suggested that I apply for a Julius Rosenwald Fellowship, helped me write the application, and of course wrote the letter of recommendation that got the Fellowship for me."

"Joe was a clear lecturer and took teaching seriously. He was proud of his ability to draw a good circle on the blackboard, using his elbow as center and his forearm as

radius. He would write on the blackboard not only formulas, statements and pictures, but also connectives *and*, *so that*, *moreover* and even *nevertheless*.''

''I regarded Joe with a mixture of admiration, affection and awe, with a desire to impress him and a desire to please him. I think that I impressed him once, and pleased him once.''

''Once several people, including Joe and me, were discussing the first lecture in a beginning probability course, explaining probability as limiting relative frequency. There was general agreement that this first lecture was not much fun, and that it was a pleasure to get on to the mathematics of probability. I then said that I never mention frequency, but spend about two minutes on an explanation that goes something like this: If there are 5 balls in a box and 2 of them are black, and you draw a ball at random, i.e. so that each ball has the same chance to be drawn, then the chance of drawing a black ball is 2 out of 5 or 2/5 or 40%. In general, for any event, we say that its chance or probability is 40% if it is just as likely as drawing a black ball from that box. On to the mathematics.

''I think that statement impressed Joe, one way or the other, because he mentioned it to me two years later.''

''I took a picture of Joe talking with Kolmogorov at the 1954 International Congress in Amsterdam. It was not a very good picture, but I sent Joe a copy. Perhaps 35 years later, I happened to be in Joe's study in Urbana. There were 3 pictures on the wall, and that picture was one of them. Clearly that picture pleased him.''

Wolfgang Haken: When I prepared this talk I expected that it would make me sad. But instead, having known Joe Doob for over 40 years is such a delightful experience that it made me happy. I hope this is acceptable.

Another problem is, of course, how to condense 40 years into 5 to 10 min, in particular, if you speak slowly. I have tried to do this by writing my talk up in as few words as I could.

I knew Joe mainly through the Saturday Afternoon Hike of which he was the Commissar for many years. Although, over the years, I even learned some Probability Theory—you can't avoid that.

One of the first things I learned about Joe was that in the 1950s he had been offered a quite distinguished professorship at MIT and that he had turned that offer down. Many colleagues were puzzled about that and he was frequently asked, ''what is so special about Urbana, Illinois, that you prefer that place to MIT at Cambridge, Massachusetts?'' Then Joe's standard reply was, ''in Urbana there is such a good hardware store.'' Many years later, Joe confided in me that in the time of the offer, his wife, Elsie, had just been diagnosed with colon cancer and that it was not clear if she would survive that (which she, fortunately, did by several decades); and under those circumstances he did not want to move. He was a very sensitive person—losing his wife and at the same time moving from the place at which he felt at home would have totally uprooted him. But for nothing in the world would he ever have said something like that.

Also, Joe found it annoying if and when colleagues bragged about their mathematical achievements and elaborated on the thought that mathematics was

the greatest thing in human culture and so on and so on. Joe had developed a quite effective, but rather mean, way of bringing such talk to an end: He asked the talker, very innocently, "how does it come that so many mathematicians go to concerts of classical music?" This was meant to encourage the talker to go on even more enthusiastically and to explain that, obviously, mathematics, as the noblest of all sciences, is closely related to the noblest of the arts... But then Joe cut him off by saying, "Oh no, it is because mathematics is so boring"—we all know that Joe was fascinated with mathematics, but he did not like this to be the centerpiece of the conversation; that was not his style.

At one of the hikes in the early 1980s, Joe told us that his son, Steve, had presented him with Minister's Certificate of the Universal Life Church and that this officially entitled him to conduct funerals and marriages. The basic beliefs of the Universal Life Church are quite simple (and thus universal) so that it is easy to become a minister there. Essentially, they believe in that which is true. Secondly, they think that it is not so easy to find out what, in detail, is true and that this should be left to the individual. I thought that this was a wonderful family where, eventually, the son finds the fitting religion for his father. However, at one of the later hikes, my daughter, Dorothea, took Joe by his word and asked him if he would agree to marry her. Joe's reply was that, most likely, his wife would object to that. But then Dorothea explained that she was asking him to conduct her marriage to her fiance, Steve Blostein, and Joe immediately agreed to that. A few weeks later, Joe conducted an excellent wedding ceremony at our house and everybody present was highly pleased.

Word about Joe's new spiritual activity was spreading and he conducted several more marriages. This went so far that, at one time our (then) Department Chairman, Philippe Tondeur, proposed to bestow on Joe the title of Chaplain of the Department of Mathematics.

Moreover, Joe gave to Dorothea and Steve a very unique and personal wedding present. This was a small pillow filled with his own hair. (He was quite skilled in cutting his hair himself and then he collected the harvest and put it to good use.) Steve and Dorothea keep this pillow as a family treasure. Perhaps, if you rest your head on it for long periods of time, some of Joe's mathematical brainpower might be transferred to you. Anyway, their son, Martin, is now 12 years old and is developing an interest in mathematics—it might really work.

Joe had asked that after he died, his ashes should be distributed at the next Saturday Afternoon Hike at one of those places where he felt that he belonged. We did that accordingly. At that hike, Nadya Shirokowa happened to be visiting and she told us a typical story about one of Joe's visits to the University of Moscow back in the time of the Soviet Union. Joe was being shown around by one of the graduate students when a security police officer stopped them and asked what Joe was doing there and if he had any document on him to identify himself. Then Joe proudly showed him his Fishing License for Lake Michigan and the somewhat puzzled officer let him go on.

Finally, another Moscow anecdote about Joe which I was told at a conference about 25 years ago. It happened in the period of time when the International

Congress of Mathematicians was taking place in Moscow and Joe was, besides being Commissar of the Saturday Afternoon Hike, President of the American Mathematical Society. After the festive presentation of the prestigious Fields Medals an excited newspaper reporter came up to Joe and exclaimed, "Professor Doob, what do you have to say about those new Fields Medallists?" And Joe replied, "Oh, I am not worried about that at all—they are as good as everybody else around here." When I asked Joe about this event, he definitely denied that he ever said anything like that. But when he understood that he reportedly said this just to a newspaper man, he softened up a bit and pointed out that he could not possibly remember what he might have said to somebody like a newspaper reporter. So, believe this at your own risk. As a friend of mine commented, "denial is not only a river in Egypt."

Naresh Jain sent to the regular hikers this email:

"I am extremely sorry that I will have to miss this important event. I know that you all are going to enjoy recalling so many happy memories of past hikes. There are two anecdotes I liked very much. Here they are:

"(1) One time Joe and Robert Kaufman were in a heated discussion about kids to be required to learn the classics. Robert was all for it and Joe was doing everything to provoke him. In disgust, Robert said: 'Oh my God', and Joe calmly replied, 'Please don't exaggerate, just call me Professor.' "

"(2) One time Joe wanted to get a pie out of me for the hikers. I was sitting there with my back toward Joe. He told everybody near him that he was going to get me into a bet which I surely would lose. He announced loudly, 'Do you guys know that Gandhi slept with two young girls all naked when he was in his late sixties?' I didn't hear him, so he repeated it again a little louder. Now this was a fact of which I was not aware, so I immediately challenged Joe and he asked if I was willing to make a bet. I did and lost. The hikers got their pie!" "

"These and many other memories will always be there in my mind and I am so thankful to all the hikers, and to Joe, in particular, for those wonderful moments. Best regards, Naresh"

Sam Wagstaff: I first met Joe in September, 1971, in the dining hall at the IAS in Princeton. He was talking with Deane Montgomery and Hassler Whitney. All three welcomed me warmly to the Institute. When I got a job offer from Illinois, I asked Joe about life in Illinois. He told me about the Saturday Hike, and I started hiking the first Saturday I lived in Illinois. I met his wife Elsie in Princeton and, later, many times at their house on High Street. Joe once led a hike in the woods at the IAS and even had a camp fire, with permission from the authorities. One nervous official came at the end of the hike and put out the fire with a fire extinguisher.

Since I have lived in Indiana, I have craved Illinois water. For many years, every Saturday I would drive to Urbana and get jugs of drinking water from Joe's place, first on High Street and then in Clark-Lindsey, and then hike. At first I would take Joe to the hike. After he retired from the hike at age 92, I would just visit him. He would often give me food, especially hot peppers, or kindling wood for the hike. Joe's house and apartment were always exciting places to visit. He had many unusual plants, including one growing upside-down.

Joe's son Steve made him an "ordained minister" of the Universal Life Church. Someone else made Elsie a "saint." Joe performed several marriages and was sometimes called the "Chaplain" of the Illinois Math Department.

Joe was Commissar of the Saturday Hike in the 1950s and/or 1960s. He created hikers delight, a dish still served today at the Hike. Let me share with you several hiking experiences I had with Joe.

Clarence Berdahl was another hiker who hiked into his nineties. One Saturday in 1980, Joe, Clarence, who was 90, and I hiked at Lodge Park. While Joe, who was only 70, and I walked around the park Clarence lit the camp fire next to a monument marking the grave of an Indian squaw. While we were eating hikers delight, a park ranger drove up and walked over to us. As he approached, Joe offered him coffee. The ranger declined and scolded us for building a fire so close to a grave stone. We promised never to do it again. He left without making us put out the fire. After he had gone, Joe told Clarence that he should not have made the fire there, and he added, "And you're old enough to know better!"

Another Saturday in the 1980s the hike went to Emerald Pool in Kickapoo Park. We parked on the road to Emerald Pool and climbed a hill to reach the camp atop a high cliff overlooking the Pool. It had a beautiful view. That night two park rangers raided our camp. The senior one walked up the gently sloping trail while the junior one climbed the steep cliff to prevent our escape that way. They told us not to have fires there again. Now we have the fire in a ravine near Emerald Pool but just outside the park boundary. Joe was disappointed that we lost this beautiful camp.

For many years Joe drove a Checker car, like a taxi, and took it on the Saturday Hike. He had several Checkers, one after the other. I recall one of them getting stuck in the snow on the way to Thomas Farm. A friendly farmer came out in his tractor and pulled Joe out of the snow drift and we continued to the hike. The Checkers could hold many passengers. The maximum number of hikers I ever heard of in Joe's Checker was 16. When Joe's last Checker was quite old, rear seat passengers could see the road go by under them through a hole in the floor.

Joe was a kind of father figure to me. After all, he attended the Ethical Culture School in New York City and graduated from its high school. He gave me much good advice on many ethical subjects.

In Spring 2004, when there was no hike for two consecutive weeks, and I saw Joe on the third week, when the hike resumed, Joe told me that,"the hike has to outlive me." Also during Spring, 2004, he asked me several times to confirm that I was the Commissar of the Saturday Hike.

I learned what he meant in June when I was asked to scatter his ashes at one of the hiking places. I chose Collison Trestle, one of Joe's favorite places to hike. His ashes are scattered near the traditional camp fire site and along the bank of the Middle Fork River for half a mile where Joe and I often walked. I dumped the remaining ashes off of the old railroad trestle into the river. According to geologist Paul Potter, it will take from 20 to 2000 years for his ashes that went into the Middle Fork to reach the Gulf of Mexico. His ashes must have been turning over when I said a prayer before scattering them.

The tiny pond in the ravine beside our camp near the Plank Bridge in Kickapoo Park has a famous history. This is what happened there on February 4, 1995:

"We came eventually to a very small, shallow pond covered with ice. Standing on the ice, I said, 'This ice will hold all of us.' I urged the others to join me to test this bet. Joe bet that I was wrong and then refused to step onto the ice, claiming that he had won the bet because the ice was not holding him." I felt that I was betting on the strength of the ice rather than on Joe's willingness to step onto the ice and that the bet could not be decided unless all five hikers stood on the ice. I eventually paid that bet with a delicious pie.

Laurie Snell: Joe completed his undergraduate work at Harvard in 1930. In an interview Doob said:

"When I graduated and it was time to think about a PhD degree, I asked Stone (Marshall) to be my advisor. He told me he had no problems for me, that I should go to J.L. Walsh, who always had many problems. Walsh accepted me and we had a fine relationship: he never bothered me, and conversely."

In 1931 Doob received his Masters degree and married Elsie Field. Doob told me that he sat next to Elsie at the Boston Symphony for 2 years before they spoke to each other. Joe completed his PhD in 1932 and then spent 2 years on a research fellowship at Columbia, spending some time also at Princeton. After that Doob could not get a job because of the Depression but found support for another year at Columbia on a grant of the statistician Harold Hotelling. So the Depression led Doob to probability.

In 1935 Doob started his academic career at the University of Illinois where he soon had three outstanding probability students: Paul Halmos, Warren Ambrose and David Blackwell. All three had been undergraduates at Illinois.

In his book *I want to be a mathematician*, **Paul Halmos** describes Doob's arrival at the University of Illinois math department.

"This boy came in looking like a graduate student, crew cut shirt sleeves and all. He was 25 years old at the time, I later learned, but he looked 19 or 20. A few days after Doob's arrival my diary starts having many entries such as: 'shoot bull with Doob,' and 'Doob's class good.' In one of our early squash games I missed a shot and swore in exasperation. 'Just call me Joe,' he said. We talked about mathematics, not to the exclusion of politics and music and professional gossip and many other human concerns, but more than about anything else."

Halmos also wrote:

"Doob has an outgoing, relaxed personality; he seems to get along easily with everyone. He seems to take nothing seriously, but he takes everyone equally seriously."

"As an example of this, Doob used to tell us that he got his first job based on a wrong theorem."

"I once asked Doob what it was like to have such great graduate students. He said 'we were all very young and a bit wild.' ".

Snell continues: I must say that in reviewing Doob's papers for this article I found them very hard to read even though I thought I knew much of the material. At a Math Colloquium at the U of I in 1997, with Doob in the audience, his friend Kai

Lai Chung remarked that one often hears that Doob's papers are hard to read. In the written account of Chung's talk we read: "Writing this suddenly put me in mind of my other experience of reading Marcel Proust's interminable recherche of his TIME. Both authors are long-winded, long-drawn-out, and palpably long-suffering, and yet both succeed in making a profound and lasting impact, and prove finally rewarding. Doob told me that he had read Proust but found him 'boring.' Some readers might say the same of Doob. Proust may be read for his style, no matter the content; Doob should be read for his content, no matter the style."

Ambrose, writing in *Halmos*, *Celebrating 50 years of Mathematics* expresses his and Paul's appreciation of Doob in a way that most of his students would appreciate: "Another topic that I wish to mention is the appreciation that Paul and I both feel to Joe Doob for what he has taught us, mainly in mathematics but also outside mathematics. Of all the people who have influenced our mathematical and other intellectual development, Joe stands out. Before Doob, neither of us had ever encountered such a forceful thinker. All our previous opinions about the world had to be modified by the serious criticism he made of them."

Snell continues: I was one of Doob's graduate students during the years 1948–1951. These were busy times for the Doobs. They had three children, and Elsie was a physician of Internal Medicine at the Carle Clinic in Urbana. The Doobs were generous with their hospitality and I was often invited to dinner. At first I could not understand why Joe and Elsie called each other thee and thou and why Joe often washed the dishes. The thee and thou was explained by the fact that Elsie came from a Quaker family. As for why Doob washed the dishes, I guess he was just ahead of his time.

At the same time Doob was writing *Stochastic Processes*, Elsie was trying to find her brothers, Noel and Hermann Field, who had disappeared behind the Iron Curtain. Doob did not believe in using secretaries. He wrote the first draft of *Stochastic Processes* using a pencil, writing on plain paper. He turned these into paper airplanes that the children used in airplane fights. When he typed (pick punch) the final version he invented a kind of miniature TeX which was highly praised by the printer.

In the preface of his *Classical Potential and Its Probabilistic Counterpart*, a book with over 840 pages, Doob thanks his typist "usually faithful, sometimes accurate." He was disappointed when he found only one person who recognized that Doob was thanking himself.

This book was supposed to be a joint book with his friend, the well-known French potential theorist Marcel Brelot. Brelot was also known for his wild driving and at a potential theory conference in France, Doob introduced Brelot by saying "I never really understood the connection between Brownian motion and potential theory until I rode with Brelot."

Joe had many chances to leave Illinois. In an interview [8] he remarks:

"I was charmed by the small-town atmosphere of Urbana as soon as I arrived and never wanted to leave, even though the atmosphere changed through the years."

His favorite example of this was the soup that a neighbor brought over when they moved into their house.

The Doobs made several trips to New Hampshire not far from where we lived. In 1959 Elsie was diagnosed with cancer. She was given the choice of 2 years to live if she had radiation treatment or 1 year otherwise. She chose the 1 year and decided to spend the summer with Joe in the mountains of New Hampshire. Joe and Elsie showed us many new places for picnics and insisted that we support all the small town theatrical groups in the area. Joe loved to lecture the restaurants on how much better real Vermont cheese was than Wisconsin cheese they were serving. Instead of 1 year Elsie lived some 30 years longer.

I visited Joe a month before he died. He was still driving and I guess I had the same feeling Joe had while driving with Brelot. However, he was his old self in every other way—anxious to talk about politics, his children, the Illinois Hike, his garden, and his latest software for his beloved Mac.

In the early time sharing computer system at Dartmouth you could type in an expression like "Edit explain life" and you got the answer "Life is a super-martingale" (an unfavorable game). I tried to think what I would put in for "Edit explain Doob". I think my wife Joan suggested the best answer: "An extraordinarily straightforward guy."

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Donald Burkholder
Department of Mathematics,
273 Altgeld Hall, 1409 West Green Street,
University of Illinois, Urbana, IL 61801, USA
E-mail address: donburk@math.uiuc.edu

Philip Protter
School of Operations Research and Industrial Engineering,
Cornell University, 220 Rhodes Hall, Ithaca, NY, 14853, USA
E-mail address: pep4@cornell.edu