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Recitation 5:

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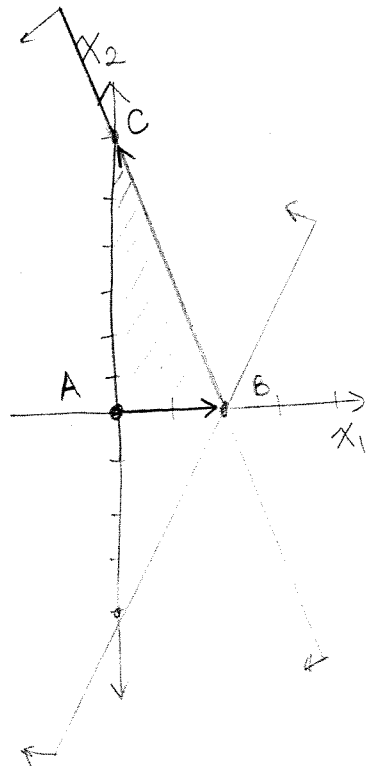
Simplex Examples:

Ex: 1

$$\begin{aligned} \min\ & -2x_1 - x_2 \\ \text{s.t.}\ & 2x_1 - x_2 + x_3 = 4 \\ & 3x_1 + x_2 + x_4 = 6 \\ & x_1,\ x_2,\ x_3,\ x_4 \geq 0. \end{aligned}$$



Simplex Tableau

		x_1	x_2	x_3	x_4	RHS
$+z$	1	$+2$	$+1$	0	0	0
x_3	0	2	-1	1	0	4
x_4	0	3	1	0	1	6
	1	0	$+2$	-1	0	-4
x_1	0	1	$-1/2$	$1/2$	0	2
x_4	0	0	$5/2$	$-3/2$	1	0 * → Deg. pivot!
	1	0	0	$+1/5$	$-4/5$	-4
x_1	0	1	0	$1/5$	$1/5$	2
x_2	0	0	1	$-3/5$	$2/5$	0
	1	-1	0	0	-1	-6
x_3	0	5	0	1	1	10
x_2	0	3	1	0	1	6

point A:

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 4 \\ 6 \end{pmatrix}, z = 0$$

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \\ 0 \\ 0 \end{pmatrix}, z = -$$

basic

point B.

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \\ 0 \\ 0 \end{pmatrix}, z =$$

basic

point C:

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 0 \\ 6 \\ 10 \\ 0 \end{pmatrix},$$

$z = -6$

Ex: 2.

$$\min \quad -4x_1 - x_2$$

$$\text{s.t.} \quad -2x_1 - x_2 + x_3 = 4$$

$$3x_1 + x_2 + x_4 = 6$$

$$x \geq 0$$

Simplex Tableau:

	$+z$	x_1	x_2	x_3	x_4	RHS
	1	+4 ↓	+1	0	0	0
x_3 ←	0	2	-1	1	0	4
x_4	0	3	1	0	1	6
	1	0	+3 ↓	-2	0	-8
x_1	0	1	-1/2	1/2	0	2
← x_4	0	0	5/2	-3/2	1	0
	1	0	0	-1/5	-6/5	-8
x_1	0	1	0	1/5	1/5	2
x_2	0	0	1	-3/5	2/5	0

$x = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \to A$

$y = \begin{pmatrix} 2 \\ 0 \\ 0 \\ 0 \end{pmatrix} \to B$ (opt. but criterio <u>NOT</u> met)

$X = \begin{pmatrix} 2 \\ 0 \\ 0 \\ 0 \end{pmatrix} \to B$ (optimally) with new basis $\{x_1, x_2\}$.

Ex ③.

$$\min\ -2x_1 - x_2$$

$$\text{s.t.}\quad \left.\begin{aligned} 2x_1 - x_2 + x_3 \quad &= 4 \\ 3x_1 - x_2 \quad + x_4 &= 6 \end{aligned}\right\} m=2$$

$$x \geq 0$$

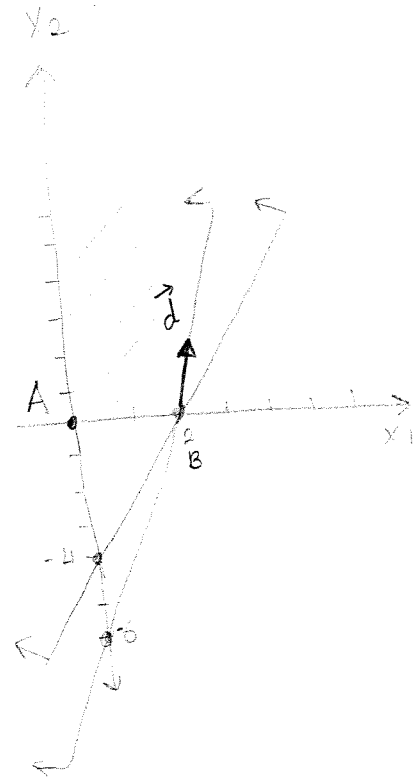
$+z$	x_1	x_2	x_3	x_4	RHS
1	$+2$ ↓	$+1$	0	0	0
← x_3	2	-1	1	0	4
x_4	3	-1	0	1	6
1	0	$+2$ ↓	-1	0	-4
x_1	1	$-1/2$	$1/2$	0	2
← x_4	0	$1/2$	$-3/2$	1	0
1	0	0	$+5$	-4	-4
x_1	1	0	-1	1	2
x_2	0	1	-3	2	0

↓ unbounded

$$x = \begin{pmatrix} 0 \\ 0 \\ 4 \\ 6 \end{pmatrix} \to \text{pt } A$$

$$x = \begin{pmatrix} 2 \\ 0 \\ 0 \\ 0 \end{pmatrix} \to \text{pt } B$$

$$x = \begin{pmatrix} 2 \\ 0 \\ 0 \\ 0 \end{pmatrix} \to \text{pt } B$$



$$d = \begin{pmatrix} 1 \\ 3 \\ 1 \\ 0 \end{pmatrix} \to \text{ray generated.}$$

Note d has $(m+1)$ pos. entries (2 corr to basic, 1 to entering non-basic) and $Ad = \begin{bmatrix} 2 & -1 & 1 & 0 \\ 3 & -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 3 \\ 1 \\ 0 \end{bmatrix} = 0$ //

as expected.