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| **Mathematical Programming** | **Homework 9** |
| **OR 630 Fall 2005** | **Due November 11, 2005** |

1. a) Suppose you are solving the standard form linear programming problem $\min\{c^T x : Ax = b,\, x \geq 0\}$. For some reason you want to apply Dantzig-Wolfe decomposition, with just one subproblem corresponding to the polyhedron $Q := \{x \in \mathbf{R}^n : x \geq 0\}$. Find what the corresponding master problem is.

b) Since that wasn't too successful, you now try to use $n$ subproblems, one for each $x_j$, with the polyhedra $Q_j := \{x_j \in \mathbf{R} : x_j \geq 0\}$. Once again, find what the corresponding master problem is.

c) Now suppose that the variables all have an upper bound of 1, so the problem becomes $\min\{c^T x : Ax = b,\, 0 \leq x \leq e\}$. As in (b), you attempt to use $n$ subproblems, one for each $x_j$, with the polyhedra $Q_j := \{x_j \in \mathbf{R} : 0 \leq x_j \leq 1\}$. Now what is the corresponding master problem?

d) Finally, consider the problem of part (c) again, but use a single subproblem, with polyhedron $Q := \{x \in \mathbf{R}^n : 0 \leq x \leq e\}$. What is the resulting master problem?

(This problem shows that you need to exercise some minimal intelligence in choosing a candidate for applying Dantzig-Wolfe decomposition!)

2. As we discussed in class, the simplex method applied to the master problem arising in Dantzig-Wolfe decomposition tends to exhibit long tails in its convergence. It would therefore be desirable to find a way to terminate the iterations with some guarantee that the current solution is within some small bound of the optimal value.

Suppose at some iteration, all the subproblems have optimal solutions. Determine a lower bound on the optimal value of the original problem, and hence a bound on how far the current solution can be from optimal.

3. Suppose you have a block-angular problem (P) as in class, with $m_0 + \sum_{j=1}^k m_j$ rows and $\sum_{j=1}^k n_j$ columns. Making the same assumptions we made in discussing the revised simplex method, compute the number of arithmetic operations (multiplications, additions, subtractions, and divisions) in one iteration of the revised simplex method applied to (P). Now consider the application of Dantzig-Wolfe decomposition. Supposing that the reoptimization of each subproblem takes $p$ iterations, and that each of the problems considered satisfies the same sparsity assumptions as before (is this reasonable?), compute the number of arithmetic operations required to do one iteration in the master problem.

4. Suppose $Q \subseteq \mathbf{R}^d$ is a polyhedron, and let $P := Q \times [0,1] := \{(x;\tau) : x \in Q,\, \tau \in [0,1]\} \subseteq \mathbf{R}^{d+1}$.

a) Show that the vertices of $P$ are of the form $(v;\tau)$ where $v$ is a vertex of $Q$ and $\tau$ is 0 or 1.

b) If $v$ and $w$ are vertices of $Q$, show that $d_P((v;0),(w;1)) = d_Q(v,w) + 1$. (You can use a very informal argument for the edges of $P$.)

c) Show that $\Delta_u(d+1, n+2) \geq \Delta_u(d,n) + 1$, and similarly for $\Delta$.