Mathematical Programming	Homework 7
OR 630 Fall 2005	Due October 28, 2005

1. Solve the following LP problem by the dual simplex method, starting with the all-surplus basis:

$$
\begin{array}{lrcrcrcrcrcrcl}
\min & 10x_1 & + & 5x_2 & + & 5x_3 & & & & & & & & \\
\text{s.t.} & 2x_1 & + & x_2 & + & x_3 & -x_4 & & & & & = & 5, \\
& x_1 & + & 3x_2 & + & x_3 & & & -x_5 & & & = & 5, \\
& x_1 & + & x_2 & + & 4x_3 & & & & & -x_6 & = & 2, \\
& & & & & & & & & & x & \geq & 0.
\end{array}
\tag{1}
$$

This is the example of the lecture notes of September 22nd., with the objective function coefficients changed to make it interesting!

Use Bland's least-index rule. Generate the information you need as required using the basis inverse, rather than using the updated equations or tableaus.

2. Consider the upper-bounded problem

$$
\begin{array}{rcrcl}
\min & & c^T x & & \\
& & Ax & = & b, \\
0 & \leq & x & \leq & u,
\end{array}
\tag{2}
$$

where A is $m \times n$ and has rank m. Suppose we write this as a standard form problem

$$
\begin{array}{lrcrcl}
\min & c^T x & & & & \\
& Ax & & & = & b, \\
(P) & x & + & w & = & u, \\
& x, & & w & \geq & 0,
\end{array}
\tag{3}
$$

with $m+n$ equality constraints. Then a basis matrix $\bar{B}$ will correspond to the choice of $m+n$ basic indices and variables. Let β consist of those j's in $\{1, 2, \ldots, n\}$ with both x_j and w_j basic; λ those j's with w_j but not x_j basic; and υ those j's with x_j but not w_j basic.

a) Show that $|\beta| = m$, and let B be the corresponding submatrix of A. Also, let N_U denote the submatrix of A corresponding to υ.

b) Write $\bar{B}$ in terms of B and N_U. For simplicity, assume that $\upsilon = \{1, 2, \ldots, k\}$, $\lambda = \{k+1, k+2, \ldots, n-m\}$, and $\beta = \{n-m+1, n-m+2, \ldots, n\}$, and that the basic variables are listed in the order x_j, $j \in \beta$, x_j, $j \in \upsilon$, w_j, $j \in \lambda$, and w_j, $j \in \beta$.

c) Obtain $\bar{B}^{-1}$ in terms of B^{-1} and N_U. (This shows how the simplex method for (P) could be efficiently implemented using only B^{-1}. But of course we can also just use the bounded-variable simplex method.)

3. Consider a 3×3 transportation problem, with all the supplies and demands equal to 1 (also known as an assignment problem), with costs given by the matrix

$$C = (c_{ij}) := \begin{bmatrix} 4 & 6 & 3 \\ 5 & 4 & 2 \\ 4 & 3 & 2 \end{bmatrix}.$$

a) Suppose the current basic variables are $x_{11}, x_{12}, x_{22}, x_{23}$, and x_{32}. Write down the equations determining the basic solution, omitting the equation for demand 3, with the rows and columns ordered so that the coefficient matrix is triangular. Hence determine the basic solution.

b) Determine the corresponding dual solution. Are the optimality conditions satisfied?

c) Draw the tree corresponding to the basic solution in (a). Suppose you want to increase x_{13}. What is the pattern of changes to the basic variables?

d) Do (a) and (b) again, where x_{13} becomes a basic variable in place of x_{12}. Comment on the differences between your answers regarding the optimality conditions.