Mathematical Programming **Homework 5**
OR 630 Fall 2005 **Due October 7, 2005**

1. The indication for unboundedness in the simplex method shows the existence of feasible solutions to the primal problem with objective function values unbounded below. This implies via weak or strong duality that the dual problem is infeasible. Show how to obtain a short certificate of the infeasibility of (D) from the quantities already computed.

2. Consider an LP problem is in the form we considered for the simplex interpretation of the simplex method:

$$\min\{c^T x : \tilde{A}x = \tilde{b},\, e^T x = 1,\, x \geq 0\},$$

where $e \in \mathbf{R}^n$ is a vector of ones. Suppose you have a basic feasible solution $\bar{x}$ for this problem, and you compute all the reduced costs $\bar{c}_N$. Show how you can obtain a lower bound on the optimal value of the problem and hence a bound on how far $\bar{x}$ is from optimality.

3. Suppose you are solving a standard form LP problem with n variables from a given basic feasible solution, and you know that every basic solution has at most one basic variable zero. Show that the simplex method will either terminate or improve the objective function value within n iterations from any basic feasible solution, and deduce that it will terminate in a finite number of iterations.