

Cycling in the Simplex Method: Example from Chvatal's Text.

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MAX      10 X1 - 57 X2 - 9 X3 - 24 X4
SUBJECT TO
    2)    0.5 X1 - 5.5 X2 - 2.5 X3 + 9 X4 <= 0
    3)    0.5 X1 - 1.5 X2 - 0.5 X3 + X4 <= 0
    4)    X1 <= 1
END

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We will run through the simplex method. The entering variable will be the one with the most negative coefficient in the objective row. The departing variable will be the candidate for departing which has the smallest subscript.

THE INITIAL TABLEAU IS:

| ROW | (BASIS) | X1 | X2 | X3 | X4 | X5 | X6 | X7 | |
|-----|---------|---------|--------|--------|--------|-------|-------|-------|-------|
| 1 | ART | -10.000 | 57.000 | 9.000 | 24.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 2 | X5 | 0.500 | -5.500 | -2.500 | 9.000 | 1.000 | 0.000 | 0.000 | 0.000 |
| 3 | X6 | 0.500 | -1.500 | -0.500 | 1.000 | 0.000 | 1.000 | 0.000 | 0.000 |
| 4 | X7 | 1.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 1.000 | 1.000 |

X1 ENTERS AT VALUE 0.000000E+00 IN ROW 2 OBJ. VALUE= 0.000000E+00

| ROW | (BASIS) | X1 | X2 | X3 | X4 | X5 | X6 | X7 | |
|-----|---------|-------|---------|---------|---------|--------|-------|-------|-------|
| 1 | ART | 0.000 | -53.000 | -41.000 | 204.000 | 20.000 | 0.000 | 0.000 | 0.000 |
| 2 | X1 | 1.000 | -11.000 | -5.000 | 18.000 | 2.000 | 0.000 | 0.000 | 0.000 |
| 3 | X6 | 0.000 | 4.000 | 2.000 | -8.000 | -1.000 | 1.000 | 0.000 | 0.000 |
| 4 | X7 | 0.000 | 11.000 | 5.000 | -18.000 | -2.000 | 0.000 | 1.000 | 1.000 |

X2 ENTERS AT VALUE 0.000000E+00 IN ROW 3 OBJ. VALUE= 0.000000E+00

| ROW | (BASIS) | X1 | X2 | X3 | X4 | X5 | X6 | X7 | |
|-----|---------|-------|-------|---------|--------|--------|--------|-------|-------|
| 1 | ART | 0.000 | 0.000 | -14.500 | 98.000 | 6.750 | 13.250 | 0.000 | 0.000 |
| 2 | X1 | 1.000 | 0.000 | 0.500 | -4.000 | -0.750 | 2.750 | 0.000 | 0.000 |
| 3 | X2 | 0.000 | 1.000 | 0.500 | -2.000 | -0.250 | 0.250 | 0.000 | 0.000 |
| 4 | X7 | 0.000 | 0.000 | -0.500 | 4.000 | 0.750 | -2.750 | 1.000 | 1.000 |

X3 ENTERS AT VALUE 0.000000E+00 IN ROW 2 OBJ. VALUE= 0.000000E+00

| ROW | (BASIS) | X1 | X2 | X3 | X4 | X5 | X6 | X7 | |
|-----|---------|--------|-------|-------|---------|---------|--------|-------|-------|
| 1 | ART | 29.000 | 0.000 | 0.000 | -18.000 | -15.000 | 93.000 | 0.000 | 0.000 |
| 2 | X3 | 2.000 | 0.000 | 1.000 | -8.000 | -1.500 | 5.500 | 0.000 | 0.000 |
| 3 | X2 | -1.000 | 1.000 | 0.000 | 2.000 | 0.500 | -2.500 | 0.000 | 0.000 |
| 4 | X7 | 1.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 1.000 | 1.000 |

X4 ENTERS AT VALUE 0.00000E+00 IN ROW 3 OBJ. VALUE= 0.00000E+00

| ROW | (BASIS) | X1 | X2 | X3 | X4 | X5 | X6 | X7 | |
|-----|---------|--------|-------|-------|-------|---------|--------|-------|-------|
| 1 | ART | 20.000 | 9.000 | 0.000 | 0.000 | -10.500 | 70.500 | 0.000 | 0.000 |
| 2 | X3 | -2.000 | 4.000 | 1.000 | 0.000 | 0.500 | -4.500 | 0.000 | 0.000 |
| 3 | X4 | -0.500 | 0.500 | 0.000 | 1.000 | 0.250 | -1.250 | 0.000 | 0.000 |
| 4 | X7 | 1.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 1.000 | 1.000 |

X5 ENTERS AT VALUE 0.00000E+00 IN ROW 2 OBJ. VALUE= 0.00000E+00

| ROW | (BASIS) | X1 | X2 | X3 | X4 | X5 | X6 | X7 | |
|-----|---------|---------|--------|--------|-------|-------|---------|-------|-------|
| 1 | ART | -22.000 | 93.000 | 21.000 | 0.000 | 0.000 | -24.000 | 0.000 | 0.000 |
| 2 | X5 | -4.000 | 8.000 | 2.000 | 0.000 | 1.000 | -9.000 | 0.000 | 0.000 |
| 3 | X4 | 0.500 | -1.500 | -0.500 | 1.000 | 0.000 | 1.000 | 0.000 | 0.000 |
| 4 | X7 | 1.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 1.000 | 1.000 |

X6 ENTERS AT VALUE 0.00000E+00 IN ROW 3 OBJ. VALUE= 0.00000E+00

| ROW | (BASIS) | X1 | X2 | X3 | X4 | X5 | X6 | X7 | |
|-----|---------|---------|--------|--------|--------|-------|-------|-------|-------|
| 1 | ART | -10.000 | 57.000 | 9.000 | 24.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 2 | X5 | 0.500 | -5.500 | -2.500 | 9.000 | 1.000 | 0.000 | 0.000 | 0.000 |
| 3 | X6 | 0.500 | -1.500 | -0.500 | 1.000 | 0.000 | 1.000 | 0.000 | 0.000 |
| 4 | X7 | 1.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 1.000 | 1.000 |

But this is the original tableau. We have cycled through 6 tableaus and returned to the initial tableau. If we continued to use the same rules for choosing the entering and departing variables then we would cycle through these tableaus indefinitely. This is called cycling and it is the only way that the simplex method can fail to terminate. Fortunately, cycling can always be avoided.

Theorem If we choose the entering and departing variables using the smallest subscript rule (as discussed in class) then the simplex method always terminates. That is, the simplex method always finds an optimal solution or shows that the problem is unbounded in a finite number of iterations. (A proof of this theorem is contained in Chvatal's text).

If we redo the last example using the smallest subscript rule then all the iterations except the last one are the same as before. However, when we get to the second to last tableau,

X1 ENTERS AT VALUE 0.00000E+00 IN ROW 3 OBJ. VALUE= 0.00000E+00

| ROW | (BASIS) | X1 | X2 | X3 | X4 | X5 | X6 | X7 | |
|-----|---------|-------|--------|--------|--------|-------|--------|-------|-------|
| 1 | ART | 0.000 | 27.000 | -1.000 | 44.000 | 0.000 | 20.000 | 0.000 | 0.000 |
| 2 | X5 | 0.000 | -4.000 | -2.000 | 8.000 | 1.000 | -1.000 | 0.000 | 0.000 |
| 3 | X1 | 1.000 | -3.000 | -1.000 | 2.000 | 0.000 | 2.000 | 0.000 | 0.000 |
| 4 | X7 | 0.000 | 3.000 | 1.000 | -2.000 | 0.000 | -2.000 | 1.000 | 1.000 |

X3 ENTERS AT VALUE 1.0000 IN ROW 4 OBJ. VALUE= 1.0000

| ROW | (BASIS) | X1 | X2 | X3 | X4 | X5 | X6 | X7 | |
|-----|---------|-------|--------|-------|--------|-------|--------|-------|-------|
| 1 | ART | 0.000 | 30.000 | 0.000 | 42.000 | 0.000 | 18.000 | 1.000 | 1.000 |
| 2 | X5 | 0.000 | 2.000 | 0.000 | 4.000 | 1.000 | -5.000 | 2.000 | 2.000 |
| 3 | X1 | 1.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 1.000 | 1.000 |
| 4 | X3 | 0.000 | 3.000 | 1.000 | -2.000 | 0.000 | -2.000 | 1.000 | 1.000 |

This tableau is optimal and cycling has been avoided.