OR 635: Interior Point Methods for Math Progg. Fall 2003.

Homework Set 3. Due: Wednesday November 19.

1. Suppose that $\phi \in SCB(C,\nu)$ with $C \subseteq \Re$, $0 \in \text{int } C$, and $\phi'(0) \geq 0, \phi''(0) = 1$. Show that

(i) For $\xi \in \text{int } C$, $\xi > 0$, $\phi'(\xi) > 0$, with $\phi'(1/4) \geq 1/6$.

(ii) If $\psi(\xi) := \phi'(\xi)$ for such ξ, then $\psi'(\xi) \geq \nu^{-1}[\psi(\xi)]^2$ and so

$$\frac{d}{d\xi}\left(-\frac{1}{\psi(\xi)}\right) \geq \frac{1}{\nu} \text{ and } -\frac{1}{\psi(\xi)} \geq -\frac{1}{\psi(1/4)} + \frac{\xi - 1/4}{\nu}.$$

(iii) $6\nu + 1/4 \notin \text{int } C$.

2. Suppose $F \in SCB(C,\nu)$ with $x \in \text{int } C$. Show that

$$\{x + h \in C : \langle F'(x), h\rangle \geq 0\} \subseteq \{x + h : ||h||_x \leq 6\nu + 1/4\}.$$

3. Suppose $F \in SCB(C,\nu)$ and C is solid and bounded. Prove that

(i) F attains its minimum at some $\bar{x} \in \text{int } C$ (called the *analytic center* of C).

(ii)

$$\{\bar{x} + h : ||h||_{\bar{x}} \leq 1\} \subseteq C \subseteq \{\bar{x} + h : ||h||_{\bar{x}} \leq 6\nu + 1/4\}.$$

(In these questions, the factor $6\nu + 1/4$ can be improved.)

4. (i) Suppose that $\phi \in SCB(C,\nu)$ with $C \subseteq \Re$, and C is of the form $[\alpha, \infty]$. Show that ϕ' is negative on int C and $\phi(\xi) \to -\infty$ as $\xi \to \infty$.

(ii) Suppose $F \in LHSCB(K,\nu)$, $\hat{x} \in \text{int } K$ and $\bar{x} \in K$. Show that $F(\hat{x} + \tau\bar{x})$ tends to $-\infty$ as $\tau \to \infty$.

(iii) Suppose F is as above, and $\{s_k\} \subseteq \text{int } K^*$ has $s_k \to s_\infty \in K^* \setminus \text{int } K^*$. Prove that $F_*(s_k) \to \infty$. (This completes the proof of the main theorem about dual barriers.)

5. Suppose you are on the central path at $(x, y, s) = (x(\mu), y(\mu), s(\mu)$. You seek the point on the central path corresponding to the parameter $\gamma\mu$, where $\gamma \in (0, 1)$. If you use the Newton equations for the nonlinear system with final equation $s + \gamma\mu F'(x) = 0$, you get the Newton step $(\Delta x_1, \Delta y_1, \Delta s_1)$, while if you replace the last equation with $x + \gamma\mu F_*'(s) = 0$, you get the Newton step $(\Delta x_2, \Delta y_2, \Delta s_2)$. Show that Δx_1 and Δx_2 are positive scalar multiples of one another, as are $(\Delta y_1, \Delta s_1)$ and $(\Delta y_2, \Delta s_2)$ (but unfortunately, not the same scalar multiples).