OR 635: Interior Point Methods for Math Progg. Fall 2003.

Homework Set 1. Due: Monday September 22.

1. Show how to generate dual estimates (y_k, s_k) during the primal affine-scaling algorithm so that, if $\{c^T x_k\}$ is bounded below, $A^T y_k + s_k = c$ for all k and $x_k^T s_k \to 0$. (This holds for either the short-step or the long-step version.) If $d_k \leq 0$, what does this imply for (y_k, s_k)?

2. Show the following properties of the primal affine-scaling method:

a) if $d_k = 0$ then x_k is optimal (in fact, if this happens at the kth iteration, it also happens at the 0th); if $d_k \geq 0$ (but not zero) then (P) is unbounded;

b) if d_k has a negative component, then x_{k+1} is well-defined and lies in $\mathcal{F}^0(P)$ (using the long-step version with $\lambda < 1$);

c) if you use the short-step version with $\alpha = 1$ (Dikin's method) and x_{k+1} is not in $\mathcal{F}^0(P)$ then it is optimal. (Hint: use Q1.)

3. Suppose $\mathcal{F}(P)$ is nonempty. Show that the following are equivalent:

a) $\mathcal{F}^0(D)$ is nonempty;

b) For all real γ, $S(\gamma) := \{x \in \mathcal{F}(P) : c^T x \leq \gamma\}$ is bounded;

c) The set of optimal solutions of (P) is nonempty and bounded; and

d) For some real γ, $S(\gamma)$ is nonempty and bounded.

(Hints: For (a) implies (b), try to replace c in the definition of $S(\gamma)$ by $\hat{s}$, where $(\hat{y}, \hat{s}) \in \mathcal{F}^0(D)$. For (d) implies (a), consider optimal dual solutions corresponding to (P) and to $\min\{(-e)^T x : Ax = b, -c^T x \geq -\gamma, x \geq 0\}$.)

4. Show that $(y(\mu), s(\mu))$ is the unique optimal solution to

$$(BD_\mu) : \max b^T y - \mu F(s),\ (y, s) \in \mathcal{F}^0(D).$$

5. Let $\phi_\rho(x; \zeta) := \rho \ln(c^T x - \zeta) + F(x)$ for $\rho \geq n, \zeta \leq v(P)$, and $x \in \mathcal{F}^0(P)$.

a) Show that if x satisfies the KKT conditions for minimizing $\phi_\rho(x; \zeta)$ over $\mathcal{F}^0(P)$, then x is on the (primal) central path.

b) Conversely, with $\rho \geq n$ fixed, show that any x on the central path satisfies these KKT conditions for some $\zeta < v(P)$.