A Short History of Interior-Point Methods

Algorithms for LP, QP, and LCP:

Author(s)	method	complexity	date
Dikin [4, 5]	affine-scaling	-	1967, '74
Karmarkar [11]	primal projective-scaling potential-reduction	$O(n\ln(1/\epsilon))$	1984
Renegar [22]	dual path-following	$O(\sqrt{n}\ln(1/\epsilon))$	1988
Gonzaga [6]	primal path-following	$O(\sqrt{n}\ln(1/\epsilon))$	1989
{ McLinden [16] { Megiddo [17]	framework for primal-dual path-following	-	1980 1989
{ Kojima-Mizuno-Yoshise [12] { Monteiro-Adler [19, 20]	primal-dual path-following	$O(\sqrt{n}\ln(1/\epsilon))$	1989
Gonzaga [7]	primal affine-scaling potential-reduction	$O(n\ln(1/\epsilon))$	1990
Ye [27]	primal-dual (asymmetric) potential-reduction	$O(\sqrt{n}\ln(1/\epsilon))$	1990
{ Todd-Ye [26] { Kojima-Mizuno-Yoshise [13]	primal-dual potential-reduction	$O(\sqrt{n}\ln(1/\epsilon))$	1990, '91.

Extensions to convex programming

Sonnevend, Jarre, Mehrotra-Sun, den Hertog [23, 10, 18, 9]	extensions to certain convex programs	1986–
Nesterov-Nemirovskii	general theory of self-concordant barriers including wide range of applications particularly SDP; book:	1988–
[21]	Interior Point Polynomial Methods in Convex Programming	1994
Alizadeh [1]	SDP: algorithms and applications to combinatorial optimization	1992

Infeasible-interior-point methods

{ Anstreicher [2] { Todd [24]	modifications of primal potential-reduction	1989–1991
{ Lustig [14] { Lustig-Marsten-Shanno [15]	modifications of primal-dual path-following	1991
Ye-Todd-Mizuno [28]	homogeneous self-dual approach	1994
Zhang [29]	complexity analysis of primal-dual path-following	1994

Survey articles: Gonzaga [8] for path-following methods; Anstreicher [3] and Todd [25] for potential-reduction methods.

References

[1] F. Alizadeh. Interior point methods in semidefinite programming with applications to combinatorial optimization. *SIAM Journal on Optimization*, 5:13–51, 1995.

[2] K. M. Anstreicher. A combined phase I – phase II scaled potential algorithm for linear programming. *Mathematical Programming*, 52:429–439, 1991.

[3] K. M. Anstreicher. Potential reduction algorithms. In T. Terlaky, ed., *Interior Point Methods in Mathematical Programming*, Kluwer Academic Publishers, Dordrecht, The Netherlands, 1996.

[4] I. I. Dikin. Iterative solution of problems of linear and quadratic programming. *Doklady Akademii Nauk SSSR*, 174:747–748, 1967. Translated in : *Soviet Mathematics Doklady*, 8:674–675, 1967.

[5] I. I. Dikin. On the convergence of an iterative process. *Upravlyaemye Sistemi*, 12:54–60, 1974. (In Russian).

[6] C. C. Gonzaga. An algorithm for solving linear programming problems in $O(n^3L)$ operations. In N. Megiddo, editor, *Progress in Mathematical Programming : Interior Point and Related Methods*, pages 1–28. Springer Verlag, New York, 1989.

[7] C. C. Gonzaga. Polynomial affine algorithms for linear programming. *Mathematical Programming*, 49:7–21, 1990.

[8] C. C. Gonzaga. Path following methods for linear programming. *SIAM Review*, 34(2):167–227, 1992.

[9] D. den Hertog. *Interior Point Approach to Linear, Quadratic and Convex Programming: Algorithms and Complexity.* Kluwer Academic Publishers, Dordrecht, The Netherlands, 1994.

[10] F. Jarre. Interior–point methods for convex programming. *Applied Mathematics and Optimization 26:287–311 (1992)*

[11] N. K. Karmarkar. A new polynomial–time algorithm for linear programming. *Combinatorica*, 4:373–395, 1984.

[12] M. Kojima, S. Mizuno, and A. Yoshise. A polynomial–time algorithm for a class of linear complementarity problems. *Mathematical Programming*, 44:1–26, 1989.

[13] M. Kojima, S. Mizuno, and A. Yoshise. An $O(\sqrt{n}L)$ iteration potential reduction algorithm for linear complementarity problems. *Mathematical Programming*, 50:331–342, 1991.

[14] I. J. Lustig. Feasibility issues in a primal–dual interior point method for linear programming. *Mathematical Programming*, 49:145–162, 1990/91.

[15] I. J. Lustig, R. E. Marsten, and D. F. Shanno. Computational experience with a primal–dual interior point method for linear programming. *Linear Algebra and Its Applications*, 152:191–222, 1991.

[16] L. McLinden. The analogue of Moreau's proximation theorem, with applications to the nonlinear complementarity problem. *Pacific Journal of Mathematics*, 88:101–161, 1980.

[17] N. Megiddo. Pathways to the optimal set in linear programming. In N. Megiddo, editor, *Progress in Mathematical Programming : Interior Point and Related Methods*, pages 131–158. Springer Verlag, New York, 1989. Identical version in : *Proceedings of the 6th Mathematical Programming Symposium of Japan, Nagoya, Japan*, pages 1–35, 1986.

[18] S. Mehrotra and J. Sun. An interior point algorithm for solving smooth convex programs based on Newton's method. In J. C. Lagarias and M. J. Todd, editors, *Mathematical Developments Arising from Linear Programming : Proceedings of a Joint Summer Research Conference held at Bowdoin College, Brunswick, Maine, USA, June/July 1988*, volume 114 of *Contemporary Mathematics*, pages 265–284. American Mathematical Society, Providence, Rhode Island, USA, 1990.

[19] R. D. C. Monteiro and I. Adler. Interior path following primal–dual algorithms : Part I : Linear programming. *Mathematical Programming*, 44:27–41, 1989.

[20] R. D. C. Monteiro and I. Adler. Interior path following primal–dual algorithms : Part II : Convex quadratic programming. *Mathematical Programming*, 44:43–66, 1989.

[21] Yu. E. Nesterov and A. S. Nemirovskii. *Interior Point Polynomial Methods in Convex Programming : Theory and Algorithms.* SIAM Publications. SIAM, Philadelphia, USA, 1994.

[22] J. Renegar. A polynomial–time algorithm, based on Newton's method, for linear programming. *Mathematical Programming*, 40:59–93, 1988.

[23] G. Sonnevend. An "analytic center" for polyhedrons and new classes of global algorithms for linear (smooth, convex) programming. In A. Prekopa, J. Szelezsan, and B. Strazicky, editors, *System Modelling and Optimization : Proceedings of the 12th IFIP–Conference held in Budapest, Hungary, September 1985*, volume 84 of *Lecture Notes in Control and Information Sciences*, pages 866–876. Springer Verlag, Berlin, West–Germany, 1986.

[24] M. J. Todd. Combining phase I and phase II in a potential reduction algorithm for linear programming. *Mathematical Programming*, 59:133–150, 1993.

[25] M. J. Todd. Potential-reduction methods in mathematical programming. *Mathematical Programming*, 76:3–45, 1996.

[26] M. J. Todd and Y. Ye. A centered projective algorithm for linear programming. *Mathematics of Operations Research*, 15:508–529, 1990.

[27] Y. Ye. An $O(n^3L)$ potential reduction algorithm for linear programming. *Mathematical Programming*, 50:239–258, 1991.

[28] Y. Ye and M. J. Todd and S. Mizuno. An $O(\sqrt{n}L)$-iteration homogeneous and self-dual linear programming algorithm. *Mathematics of Operations Research*, 19:53–67, 1994.

[29] Y. Zhang. On the convergence of a class of infeasible interior–point methods for the horizontal linear complementarity problem. *SIAM Journal on Optimization*, 4:208–227, 1994.