History of Mathematical Programming in the USSR: Analyzing the Phenomenon

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Mathematics in the USSR: reasons for success

- Government support; military demand

- High level of education; mathematical circles and Olympiads

- Research institutions

- Cheap books

- Lack of other career ways for active young people

Mathematics in the USSR: back side

- Obstacles in international contacts
- Party and authorities' intervention in scientific life
- Mania of secrecy
- Anti-Semitism and other restrictions
- Monopolization of science

Prehistory: 18th century

L.Euler, 1707–1783; lived in Russia 1727–1741 and 1766–1783; published more than 850 papers and books.

"There is nothing in the world where we can not see a sense of maximum or minimum; thus there is no doubt that all phenomena in the nature can be explained via maximum and minimum method..."

- Higher order necessary and sufficient optimality conditions for unconstrained optimization

- Calculus of variations: Euler equation , Discrete approximation, Isoperimetric problems

Prehistory: 1800's-1930's

P.L.Chebyshev, 1821–1894 — Chebyshev approximation.

$\min_x \max_{t\in T} |a(t) - \sum_i x_i f_i(t)|$

"...the same problem is common for all practical activity of human beings: how to allocate our resources for achieving the profit as great as possible? Solving of such problems constitutes the subject of so called theory of maximal and minimal quantities. These problems, of purely practical origin, have a special significance for theory as well: all laws, governing the movement of weighted or weightless matter, are the solutions of problems of such sort. We can not miss their productive influence on the development of mathematical sciences".

A.A.Markov, 1856–1922 — problem of moments.

min $\int_a^b t^n f(t)dt, \quad 0 \leq f(t) \leq L, \quad \int_a^b t^i f(t)dt = c_i, \quad i = 1, \ldots, n-1$

A.M.Lyapunov, 1857–1918 — stability theory

$\dot{x} = f(x)$

Solution $x(t)$ is stable if there exists $V(x)$ (Lyapunov function) such that

$$(\nabla V(x), f(x)) < 0$$

Reversed point of view: the differential equation is a continuous-time method for minimization of $V(x)$.

The Pioneer

L.V.Kantorovich, 1912–1986 Graduated Leningr. University at 18, full Professor at 22, first paper published at 16, Stalin prize, 1949, Lenin prize, 1965, Nobel prize, 1975.

Three breakthroughs in optimization

- Linear programming 1939
- General optimality conditions 1940
- Functional analysis techniques 1944–1948

Linear Programming

Book: L.V.Kantorovich, *Mathematical methods for organization and planning of production*, Leningrad University Publ., 1939, 67 pp.

- New type of optimization problems addressed

- Many applications indicated

- Ideas of numerical methods presented, based on dual variables ("resolving factors")

  However, the revolutionary book caused little respond from economists or mathematicians! Reasons:

1) no demand for mathematical methods in totalitarian system

2) the work has not been written as a mathematical text

Kantorovich returned to LP much later. Second book:

L.V.Kantorovich, *Economical calculation of the best utilization of resources*, Academy of Science Publ., 1960, 347 pp.

It contains two mathematical Appendices; but the main efforts are paid to prove compatibility with Marxist dogmata. For instance, dual variables are called not “prices”, but “objective-conditioned estimates” (“o.o.o.” in Russian).

General optimality conditions

L.V.Kantorovich, On an effective method for solving some classes of extremum problems, *Doklady AN SSSR*, 1940, 28, No. 3, pp. 212–215.

$$\min_{x \in Q} f(x)$$

$f(x)$ is a quasiconvex functional on a Banach space X, $Q \subset X$ is a convex set + minor technical assumptions.

Theorem x^* is a minimum point if and only if there exists $h \in B^* : h(x) \leq h(x^*) \quad \forall x \in Q, h(x) \geq h(x^*) \quad \forall x : f(x) \leq f(x^*)$.

Applications to constrained optimization of integral functionals and to solving inconsistent equations are given. Surprisingly, this brilliant mathematical paper also caused no respond! Probably it advanced beyond its time...

Functional analysis techniques

L.V.Kantorovich, Functional analysis and applied mathematics, *Uspekhi Matem. Nauk (Russian Math. Surveys)*, 1948, 3, No. 6, pp. 89–185.

- Convergence of the steepest descent method

- Convergence of Newton method

- Use of functional analysis techniques for optimization problems

This paper established high level standards for validation of numerical methods of optimization.

Fifties: emerging new science

E.Ya.Remez, 1957 (1935) — algorithms for Chebyshev approximations

S.I.Zukhovitskii, 1951 — Doctoral thesis "Some problems of Chebyshev approximations" — finite methods for solving linear inequalities and finding best Chebyshev approximations for inconsistent linear equations

S.N.Chernikov, 1953 — systems of linear inequalities

M.G.Krein, 1953 — extensions of Markov's problem of moments

G.S.Rubinstein, 1955 — rigorous mathematical formulation of LP problems and their analysis

L.S.Pontryagin, 1956 — maximum principle for optimal control

First book on LP in Russian, 1959 — translation of *Linear inequalities and related systems*, H.Kuhn, A.Tucker (eds)

E.S.Ventzel, 1959 — first Russian book on matrix games

D.B.Yudin, E.G.Golstein, 1961 — first Russian book on LP

Sixties: golden age

The time was ripe. Avalanche of research. Main centers:

- Moscow: Moscow State University (Girsanov, Tikhomirov, B.Polyak, Vasiliev), Yudin's Laboratory (Yudin, Golstein, later Ioffe, Nemirovskii), Central Economical - Mathematical Institute (Golstein, later Skokov, Tretyakov, Nesterov), Computer Center of the Academy of Sciences (Moiseev, Evtushenko, later Khachiyan, Antipin), other institutes (Dubovitskii, Miljutin).

- Kiev: Glushkov Institute of Cybernetics (Ermoliev, Pshenichnyi, Shor, Mikhalevich, Nurminskii), other institutes (Zukhovitskii, R.Polyak, Primak).

- Novosibirsk: Institute of Mathematics (Kantorovich, Rubinstein, Dikin, Bulavskii, Rubinov, Kaplan).

- Leningrad: Leningrad State University (Demyanov, Romanovskii, Vershik).

- Other cities: Kharkov (Ljubich, Majstrovskii), Sverdlovsk (Eremin), Irkutsk (Bulatov), Minsk (Gabasov, Kirillova, later Mordukhovich), Voronezh (Krasnoselskii, Levin) etc.

Sixties: main journals

- Zhurn. Vychisl. Matem. i Matem. Phys. = USSR Journ. Comp. Math. and Math Phys. — Moscow, journal on numerical analysis

- Kibernetika = Cybernetics — Kiev, journal on computer science, system theory and optimization

- Ekonomica i Matem. Metody = Mathecon — Moscow, journal on mathematical economics

- Avtomatika i Telemekh. = Automation and Remote Control — Moscow, journal on control

- Doklady AN SSSR = Soviet Math. Doklady — Moscow, all branches of science

However, we never had a special journal on MP!

Sixties: main directions of research

- General theory of extremum problems — can be traced to Kantorovich,1940
  - Dubovitskii-Miljutin, 1963 — cone techniques for general necessary conditions
  - I.V.Girsanov, *Lectures on mathematical theory of extremum problems*, Moscow State Univ. Publ., 1970
  - B.N.Pshenicnyj, *Necessary conditions for extremum,* Nauka, 1969
  - E.G.Golstein, *Duality theory in mathematical programming and its applications*, Nauka, 1971
  - A.D. Ioffe, V.M.Tikhomirov, *Theory of extremum problems*, Nauka, 1974

- Numerical methods for general extremum problems — originate from Kantorovich, 1948
  - E.S.Levitin, B.T.Polyak, *Minimization methods under constraints*, 1966
  - V.F.Demyanov, A.M.Rubinov, *Approximate methods for solving extremum problems*, Leningr. State Univ. Publ., 1968
  - B.N.Pshenichnyj, Yu.M.Danilin, *Numerical methods in extremum problems*, Nauka, 1975
  - F.P.Vasiliev, *Lectures on methods for solution of extremum problems*, Moscow State Univ. Publ., 1974
  - B.T.Polyak, *Introduction to optimization*, Nauka, 1983

- Nonsmooth optimization

  - Shor, 1962, 1970; Ermoliev, 1966; Polyak, 1967, 1969; Eremin 1965, 1967 — subgradient method and its extensions; summarized in N.Z.Shor, *Methods of minimization of nondifferentiable functions and their applications*, Nukova Dumka, 1979

  - A.Yu.Levin, 1965 — center of gravity method

  - Minimax problems — V.F.Demyanov, V.M. Malozemov *Introduction to minimax*, Nauka, 1972

- Stochastic optimization

- Yu.M.Ermoliev, *Stochastic programming methods*, Nauka, 1976
- D.B.Yudin, *Problems and methods of stochastic programming*, Nauka, 1979

- Linear programming and related topics
  - Iterative methods — Dikin, 1967; Polyak - Tretyakov, 1972
  - Decomposition methods — Golstein
  - Textbooks — Yudin-Golstein, 1961, 1963, 1966; Zukhovitskii - Avdeeva, 1967
  - Software — Romanovskii, Malkov, Stanevichus, Kim, Skokov and many others

- Discrete optimization — Finkelstein, Korbut, Fridman, Levner, Sigal, Sergienko; see A.A.Korbut, Yu.Yu.Finkelstein, *Discrete programming*, Nauka, 1969

- Applications — Girsanov (chemical processes, industrial planning). But in general there were few successful applications of optimization techniques in real-life problems in USSR.

Sixties: Main Events

- Seminars and lecture courses on optimization, MGU, 1962 — Girsanov, Tikhomirov, B.Polyak

- Seminars and lectures in Kiev, 1960 — Zukhovitskii, Pshenychnyi, Shor

- International Congress of Mathematicians, Moscow, 1966 — first contact with Western experts

- Drogobych Winter Schools on MP, 1968-1975 — chaired by S.Zukhovitskii

- Summer Schools on Optimization, — chaired by N.Moiseeev

- All-union Symposia on Optimal Programming Software, — chaired by E.Golstein

- All-Union Conferences on Mathematical Programming

- All-Union Symposia on Extremum problems

- All-Union Conferences on Numerical Analysis

1970–80's: New Directions

- Complexity of optimization problems Yudin - Nemirovskii 1979

- Effective methods — Yudin - Nemirovskii 1979, Nesterov 1989

- Method of ellipsoids and polynomial complexity of LP — Yudin - Nemirovskii 1976, Shor 1977, Khachiyan 1979

- Interior-point polynomial methods — Dikin 1967, Nesterov - Nemirovskii 1989-1994

- Semidefinite programming — Nesterov - Nemirovskii, 1989-1994

- New progress in convex and nonsmooth analysis — Mordukhovich, Ioffe

Sad view on contemporary situation

- Few people of MP community remain in the country...

- Lack of seminars, conferences, workshops inside the country and difficulties in visiting such meetings abroad...

- No young people starting their research...

End of history?

No USSR, but the history continues?