ORIE 6334 Bridging Continuous and Discrete Optimization

Course Information

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1 Instructor Information

Instructor:	David Williamson
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Office hours:	Tuesdays 1:30-2:30, Fridays 11-12, and by appointment.
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2 Lectures

Lectures will be held in Upson 206 on Mondays and Wednesdays from 11:40AM-12:55PM. Lectures will be broadcast to Cornell NYC Tech, Bloomberg 091.

3 Course content

This course will consider the interplay between continuous and discrete optimization broadly speaking, but with a focus on algorithmic spectral graph theory and applications of the multiplicative weights update paradigm. Within algorithmic spectral graph theory, both older structural results and recent algorithmic results will be presented. Topics to be covered include the matrix-tree theorem, Cheeger's inequality, Trevisan's max cut algorithm, bounds on random walks, Laplacian solvers, electrical flow and its applications to max flow, spectral sparsifiers, and the Colin de Verdiere invariant. Additional topics may include the Arora-Rao-Vazirani algorithm for sparsest cut, online algorithms for set cover and other problems, and results on maximizing submodular function maximization under various types of constraints.

4 Course website

The course website will be at `www.orie.cornell.edu/~dpw/orie6334/index.html`. Various materials from the course will be posted there.

5 Prerequisites

There is no formal prerequisite. In practice, I will be assuming some previous exposure to linear algebra, and either to algorithms or combinatorial optimization, and some ability to do mathematical proofs. If you've had some linear algebra and a good undergraduate algorithms class that had proofs about the algorithms, you should be set. Please talk to me if you have questions about whether you have the necessary background.

6 Course materials

There is no required textbook. Because there is no required text, we will use a system to which I was subjected to in graduate school and which worked reasonably well. Each week an official "scribe" will be appointed to take notes for that day's class. The scribe will then get to me a clearly written version of the notes (preferably in LaTeX) by within six days, so that I can post these notes within a week. Producing scribe notes will be a requirement of the course, even for auditors. The number of times a student will need to scribe will depend on how many students end up taking the course. In some cases, scribe notes from a previous year will be available, and the scribe need only edit the notes to reflect what was taught that day.

There have been a number of excellent courses taught elsewhere on this topic, and I will be drawing heavily on the lecture notes posted from these classes. I will point out where I am drawing my material from as the course proceeds. Some of these resources are listed below.

- David P. Williamson, Cornell University, ORIE 6334, Algorithmic Spectral Graph Theory, Fall 2016, `https://people.orie.cornell.edu/dpw/orie6334/Fall2016`.
- Lap Chi Lau, University of Waterloo, CS 798, Algorithmic Spectral Graph Theory, Fall 2015, `https://cs.uwaterloo.ca/~lapchi/cs798/-2015`.
- Lap Chi Lau, University of Waterloo, CS 860, Spectral Graph Theory, Spring 2019, `https://cs.uwaterloo.ca/~lapchi/cs798/-2015`.
- Luca Trevisan, UC Berkeley, Lecture Notes on Expansion, Sparsest Cut, and Spectral Graph Theory, `https://people.eecs.berkeley.edu/~luca/books/expanders.pdf`.
- Dan Spielman, Yale University, CS 662, Spectral Graph Theory, Fall 2015, `http://www.cs.yale.edu/homes/spielman/561/`.
- Dan Spielman, Yale University, CS 662, Spectral Graph Theory, Fall 2018, `http://www.cs.yale.edu/homes/spielman/561/syllabus.html`.
- Nisheeth Vishnoi, EPFL, $Lx = b$, `https://theory.epfl.ch/vishnoi/Lxb-Web.pdf`.
- Chris Godsil and Gordon Royle, *Algebraic Graph Theory*, Springer, 2001. `http://link.springer.com/book/10.1007%2F978-1-4613-0163-9`.

7 Requirements

There will be somewhere between 4-5 problem sets, handed out and collected on a bi-weekly basis. There will be a choice of either a final project or a final takehome exam; the exam/project will count for somewhat more than a problem set. Examples of a final project include writing a survey of material not covered in the class, or an implementation of some of the algorithms covered in class. Scribing once or twice will also be a small portion of the final grade.

8 Collaboration

Cornell's Code of Academic Integrity can be found at `cuinfo.cornell.edu/aic.cfm`.

Your work on problem sets and exams should be your own. You may discuss approaches to problems with other students, but as a general guideline, such discussions may not involve taking notes. You must write up solutions on your own independently, and acknowledge anyone with whom you discussed the problem by writing their names on your problem set. You may **not** use papers or books or other sources (e.g. material from the web) to help obtain your solution. Use of such materials will be considered an academic integrity violation.

No collaboration will be allowed for the take-home final.

9 Schedule

Here is an extremely rough schedule for the first part course, which is subject to change without notice.

- Course intro: spectral properties, the multiplicative weight update algorithm (2 weeks)
- Eigenvalue/eigenvector basics and identities (1 week)
- Graph Laplacians, some properties, matrix-tree theorem (1 week)
- Cheeger's inequality and analogs. Trevisan's algorithm for MAX CUT. (1 weeks)
- Generalized Laplacians and the Colin de Verdiere invariant (1 week)
- Electrical flow basics, Laplacian solvers, connections to max flow (3 weeks)
- Spectral sparsifiers (1 week)

Topics to be covered at the end of the semester include the Arora-Rao-Vazirani approximation algorithm for sparsest cut and connections to eigenvalue bounds, submodular function maximization subject to constraints, and other results.