ORIE 6300 Mathematical Programming I October 10, 2014

Problem Set 7

Due Date: October 17, 2014

1. (10 points) In class, we started to describe the capacitated simplex method for solving linear programs of the form $\min c^T x : Ax = b, \ell \leq x \leq u$. We noted that its dual is $\max b^T y - u^T v + \ell^T w : A^T y - v + w = c$, and that for any y, we can have a dual feasible solution by setting $v = \max(0, A^T y - c)$ and $w = \max(0, c - A^T y)$. We stated that the main idea is now to maintain three sets of indices of the primal variables: B the basic variables, L the variables set to their lower bounds, and U the variables set to the upper bounds, so that associated with these sets, we have a solution x in which we set $x_j = \ell_j$ for all $j \in L$, $x_j = u_j$ for all $j \in U$, and $x_B = A_B^{-1}(b - A_U u_U - A_L \ell_L)$; we assume the primal is feasible, which is true if $\ell_B \leq x_B \leq u_B$.

   Given the dual solution $y = (A_B^T)^{-1} c_B$, and the normal reduced costs $\bar{c} = c - A^T y$, we argued in class that the current primal and dual are optimal if $\bar{c}_j \geq 0$ for all $j \in L$ and $\bar{c}_j \leq 0$ for all $j \in U$. Finish the description of the simplex method by describing what should happen from this point on: if the solutions are not optimal, how should x, B, L, and U be altered so that x remains feasible and so that we make progress if the current solution is not degenerate? How do we know that the updated B is a basis?