ORIE 6300 Mathematical Programming I September 5, 2014

Problem Set 2

Due Date: September 12, 2014

1. Give an example of a primal-dual pair for which both the primal and dual are infeasible, and demonstrate that they are infeasible. Use a matrix $A \in \Re^{1\times 1}$.

2. Suppose that $P^i = \{x \geq 0 : A^i x = b^i\}$ for $i = 1, 2$ are both bounded.

   Prove that $P = P^1 + P^2$ is also a polytope, where $P^1 + P^2 = \{x^1 + x^2 : x^1 \in P^1 \text{ and } x^2 \in P^2\}$.

3. Suppose that you are given a feasible solution $\bar{x}$ of value $\bar{\gamma}$ to the problem $\max(c^T x : Ax \leq b)$. Give a method that either demonstrates that the feasible region is unbounded (i.e., there is a point x and direction y such that $x + \lambda y$ is feasible for all $\lambda > 0$) or that finds a vertex $\tilde{x}$ of the feasible region with objective value $c\tilde{x} \geq \bar{\gamma}$. Your method should not use general purpose linear programming algorithms (like the simplex method). (Hint: some of the discussion of the equivalence of bounded polyhedra and polytopes, as well as the equivalence of extreme points, vertices, and basic feasible solutions, might prove useful.)