Large-Network Travel Time Distribution Estimation for Ambulances

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Abstract

We propose a regression approach for estimating the distribution of ambulance travel times between any two locations in a road network. Our method uses ambulance location data that can be sparse in both time and network coverage, such as Global Positioning System data. Estimates depend on the path traveled and on explanatory variables such as the time of day and day of week. By modeling at the trip level, we account for dependence between travel times on individual road segments. Our method is parsimonious and computationally tractable for large road networks. We apply our method to estimate ambulance travel time distributions in Toronto, providing improved estimates compared to a recently published method and a commercial software package. We also demonstrate our method’s impact on ambulance fleet management decisions, showing substantial differences between our method and the recently published method in the choice of which ambulance to send to an emergency and in the predicted probability that an ambulance arrives within a time threshold.

Keywords: Transportation; Traffic; OR in health services; Travel time estimation; Markov Chain Monte Carlo.

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1 Introduction

Estimates of ambulance travel times on arbitrary routes in a road network are used in ambulance dispatch decisions, base location algorithms, and real-time redeployment methods [1, 6, 13]. In many of these applications it is also important to capture the uncertainty in the travel time, by predicting the entire travel time distribution rather than just the expected travel time [13]. For instance, taking into account uncertainty in the travel time of ambulances to the scene of an emergency can substantially increase the survival rate of cardiac patients, by improving fleet management decisions and thus reducing response times [7, 21]. Also, ambulance fleet performance is measured by the fraction of emergency calls for which the response time is less than a specified threshold, and forecasting this performance measure requires travel time distribution information [19].

We propose a regression approach for estimating the distribution of an ambulance travel time on an arbitrary route in a road network. The prediction depends on the route and on explanatory variables such as the time of day and day of week. Our method uses information from historical trips on the network, specifically the total travel time and estimated route for each trip. In order to predict the travel time distribution for a particular route, we do not require historical trips that take precisely the same route. Instead, our statistical approach uses information from all the historical trips by learning shared properties like the effects of time of day and types of road traversed. The model we use is intuitive and its parameters are interpretable. Our method is computationally efficient, scaling effectively to large road networks and large historical trip databases.

Two features of ambulance travel times motivate our modeling choices. First, ambulances traveling at “lights-and-sirens” speeds are less affected by traffic than other vehicles (see Section 5.3). Therefore, historical ambulance trips are the most relevant source of information for travel times, and real-time traffic flow information from other vehicles is less useful. Second, ambulance trips are comparatively rare, implying that ambulance data is sparse in time and road network coverage. Roads that are not major thoroughfares may have only a few ambulance trips on them per year.

To estimate the route taken in the historical trips, we use Global Positioning System (GPS) measurements taken during travel. This source of data is called floating car data or automatic vehicle location data, and is increasingly available for many types of vehicles, including ambulances, taxis, and personal vehicles, via GPS-enabled smartphones or 2-way navigation devices.
(e.g. Garmin or TomTom) [11]. Unlike other sources of travel time data, it does not require instrumentation on the roadway, and thus is the only source of data available to estimate travel times that has the prospect of comprehensive network coverage.

Despite the rise in availability of floating car data, there are still few methods available to utilize floating car data for travel time distribution prediction. Hofleitner, Herring, and Bayen [12] and Hofleitner, Herring, Abbeel, and Bayen [11] take a traffic flow perspective, modeling at the level of the network link (a road segment between two intersections). They use a dynamic Bayesian network for the unobserved traffic conditions on links and model the link travel time distributions conditional on the traffic state. Their method is applied to a subset of the San Francisco road network with roughly 800 links, predicting travel times using taxi fleet data and validating with additional data sources.

In previous work, we introduced a Bayesian model for simultaneous travel time distribution and path estimation for a set of vehicle trips [29]. Like Hofleitner et al., we modeled travel times at the link level. Our method was applied to estimate ambulance travel times on a subregion of Toronto.

Jenelius and Koutsopoulos [14] propose a framework for estimating the distribution of travel times while incorporating weather, speed limit, and other explanatory factors. They point out that previous approaches such as Hofleitner et al. and Westgate et al. [11, 12, 29] assume that the link travel times are independent within a vehicle trip, perhaps conditional on the traffic state. This contrasts with empirical evidence suggesting that the link travel times are strongly correlated, even after conditioning on time of day and other explanatory factors [3, 24]. Therefore, they capture correlation using a moving average specification for the link travel times. Their framework is applied to estimate travel times for a particular route in Stockholm.

In contrast to these approaches, we model travel times at the trip level instead of the link level. This naturally incorporates dependence between link travel times. The ambulance route is taken into account in the specification of the trip travel time parameters, such as the median travel time. This trip-level approach is related to the regression approach of Budge, Ingolfsson, and Zerom [5], who model the travel time distribution for an ambulance trip as a function of shortest-path distance between the start and end locations. They assume that the log travel time follows a \( t \)-distribution, where the centering and scale parameters are either a nonparametric or parametric function of the shortest-path distance. These functional forms enable their method to be flexible but still interpretable. Like them we take a regression approach, but we also incorporate dependence on the route taken, time of day, and other explanatory factors, justifying
our modeling choices empirically. This captures the fact that locations near primary roads can be reached more quickly than other locations, for example. A downside of modeling at the trip level is that travel time predictions cannot be updated to reflect changing conditions while a vehicle is enroute. However, this is not a drawback in the ambulance setting, because travel time estimates are used for ambulance dispatch decisions and base placement, rather than for route selection.

We use our method to predict ambulance travel times for the entire road network of Toronto. The size of the road network (68,272 links) is an order of magnitude larger than in previous applications of travel time distribution estimation based on floating car data [5, 11, 12, 14, 29], and the number of historical vehicle trips (157,283) is also larger than these previous applications. We compare the prediction accuracy of our method to that of Budge et al. [5], Westgate et al. [29], and a commercial software package for mean travel time estimation. We also consider the effect of various simplifications of our model, and investigate the accuracy of our model when the time effect on travel times is artificially inflated.

Finally, we evaluate the effect of using our method for ambulance fleet management, relative to that of Budge et al. [5]. We do this by selecting a set of representative ambulance posts in Toronto. We calculate which ambulance post is estimated to be the closest in median travel time to each intersection in Toronto, and find that many intersections have different estimated closest posts, according to the two methods. Therefore, the two methods would recommend that a different ambulance respond to emergencies at these locations, if the closest ambulance is dispatched. We also calculate the probability that an ambulance is able to respond on time (within a specified time threshold) from the closest post to each intersection of the city. We find substantial differences in these probabilities between the two methods. These appear to arise because our method captures differences in speeds between different types of roads, unlike the method of Budge et al.

Commercially-available vehicle travel time estimates typically consist of estimated expected travel times rather than distribution estimates, so they cannot be used to calculate the probability an ambulance arrives within a time threshold, or for ambulance deployment algorithms where simulated travel times are needed. Also, these estimates are calculated for standard vehicle speeds, not “lights-and-sirens” ambulance speeds. However, they are still useful for point estimation performance comparisons, as long as they are corrected for bias. Specifically, we investigate travel time estimates from TomTom, a maker of navigation devices.

The article is organized as follows. In Section 2 we introduce our statistical model and esti-
mation method. In Section 3, we introduce the data from Toronto and highlight the exploratory data analysis that motivates our modeling choices. In Section 4, we give details on the estimates from TomTom. In Section 5, we discuss estimation results and comparisons with the alternative methods. We draw conclusions in Section 6. In Appendices A and B, we give details on data preprocessing and fastest path estimation.

2 Modeling and Estimation

2.1 Travel Time Modeling

Consider a road network with links indexed by \( j \in \{1, \ldots, J \} \) and a set of vehicle trips on that network indexed by \( i \in \{1, \ldots, I \} \). Let \( d_j \) indicate the length of link \( j \). Assume that each trip \( i \) begins and ends at known locations on the road network (not necessarily at intersections), and that the sequence of links \( A_i = \{A_{i1}, \ldots, A_{in_i} \} \) traversed by trip \( i \) is known. Let \( f_{ij} \) denote the known fraction of link \( j \) used by trip \( i \). For interior links in the path \( A_i \), this fraction equals 1; for the first and last links, it captures the fraction of the link actually traversed during the trip.

The travel time \( T_i \) for trip \( i \) is modeled with a lognormal distribution, conditional on the route traveled. Specifically,

\[
T_i | A_i, \{f_{ij}\}_{j \in A_i}, \{d_j\}_{j \in A_i} \sim \mathcal{LN} \left( \mu(i) + \log \left( c + \sum_{j \in A_i} f_{ij}d_j u(i,j) \right), \sigma^2(i) \right)
\]

conditionally independent across trips \( i \), where the functional forms of \( \mu(i) \), \( u(i,j) \), and \( \sigma^2(i) \) are specified appropriately for the context. This model can be rewritten as \( T_i = R_i(c + \sum_{j \in A_i} f_{ij}d_j u(i,j)) \) for a random lognormal multiplicative factor \( R_i \sim \mathcal{LN}(\mu(i), \sigma^2(i)) \) capturing the travel time variability and trip-level effects. The baseline travel time is given by \( (c + \sum_{j \in A_i} f_{ij}d_j u(i,j)) \), where the term \( u(i,j) \) is a unit travel time (inverse of speed) for trip \( i \) on link \( j \). The product \( f_{ij}d_j \) is the distance traveled on link \( j \) in trip \( i \), so the baseline travel time is a sum of individual link travel times plus an intercept \( c > 0 \). One can also include intersection and turn effects in the specification; we do not focus on this extension because it has a negligible effect on predictive accuracy in our application to ambulance travel times, since ambulances do not have to obey traffic signals when traveling at lights-and-sirens speeds.

The intercept \( c \) captures, for instance, additional time required to get up to speed at the beginning of the trip and to slow down at the end. Its inclusion is similar to the model introduced
by Kolesar et al. [17] and used by Budge et al. [5], in which the travel times depend on the square root of the distance for small distances, and grow linearly with the distance for large distances.

The unit travel time \( u(i, j) \) for link \( j \) in trip \( i \) can depend on explanatory factors like the road class, speed limit, and whether the road is one-way. Additionally, it can depend on the type of vehicle or the driver. Most simply it can be a link effect, giving the form \( u(i, j) \triangleq u_j \). However, if there are links with very few trips, as is the case for ambulance data, this approach yields noisy estimates of the \( u_j \) parameters. For the ambulance study, we specify \( u(i, j) \) to depend on the road class, taking \( u(i, j) \triangleq u_{\ell(j)} \) where \( \ell(j) \in \{1, \ldots, L\} \) is the road class of link \( j \) (highway, arterial road, etc.). One could also partition the road network into \( R \) geographic regions, and take \( u(i, j) \triangleq u_{\ell(j), r(j)} \) for \( r(j) \in \{1, \ldots, R\} \), to allow downtown arterial roads to be distinguished from suburban arterial roads, for example.

The parameters \( \mu(i) \) and \( \sigma^2(i) \) for the trip effect can depend on time, weather, driver, and other explanatory factors (similarly to Jenelius et al. [14]). For the ambulance study, we use time bin as an explanatory factor, setting \( \mu(i) \triangleq \mu_k(i) \) where the week is divided into time bins \( k \in \{0, 1, \ldots, K\} \) and \( \mu_0 \triangleq 0 \) to ensure model identifiability, i.e. so that each parameter of the model can be uniquely determined given sufficient data.

For the ambulance study, we specify the log scale variance \( \sigma^2(i) \) using exponential decay in the total trip distance \( d_i \triangleq \sum_{j \in A_i} f_{ij} d_j \), as suggested by exploratory data analysis (see Section 3.1, Figure 1). Specifically, we take \( \sigma^2(i) \triangleq Me^{-\lambda d_i} + \delta \), for parameters \( M > 0, \lambda > 0 \), and \( \delta > 0 \). With this choice, the variance of the log travel times approaches \( \delta \) as the trip distance increases, and equals \( M + \delta \) for trips of length zero. The parameter \( \lambda \) controls how quickly the variability decreases towards \( \delta \). For our ambulance study, the unknown parameters in the model are then \( \theta \triangleq (c, u_1, \ldots, u_L, \mu_1, \ldots, \mu_K, M, \delta, \lambda) \).

### 2.2 Estimation

We use a Bayesian formulation to estimate the model parameters, which allows uncertainty in the parameter estimates to be taken into account for travel time predictions. The predictions are based on the posterior distribution of the parameters, which is proportional to the prior density (specified below) times the likelihood function. The likelihood function is equal to the product over trips \( i \) of the lognormal density of \( T_i \) as specified in (1). We estimate each parameter and relevant function of the parameters by its posterior mean, and summarize our uncertainty with a 95% interval estimate, the endpoints of which are the 0.025 and 0.975 quantiles of the posterior
distribution. Computation of the posterior distribution is done via Markov chain Monte Carlo [27].

For our ambulance application, we have found results to be robust to moderate changes in the prior distributions for the unknown parameters \((c, u_1, \ldots, u_L, \mu_1, \ldots, \mu_K, M, \delta, \lambda)\), due to the large volume of data. Results are reported for the following prior distributions, with mutually independent parameters:

\[
\begin{align*}
u_\ell &\sim LN(\nu_\ell, \xi^2_u), \quad \ell \in \{1, \ldots, L\}, \quad k \in \{1, \ldots, K\} \\
c &\sim Unif(0, \infty), \\
\sqrt{M} &\sim Unif(0, \infty), \\
\sqrt{\delta} &\sim Unif(0, \infty), \\
\lambda &\sim Unif(0, \infty). 
\end{align*}
\]

The constant \(\nu_\ell\) is a prior estimate of the unit travel time on the log scale, for road class \(\ell\). For example, there might be initial speed estimates for each link in class \(\ell\), or perhaps known speed limits or recorded GPS speed data. In such cases, \(\nu_\ell\) can be set equal to the mean of the log inverse speeds. For the ambulance study, we use GPS speed data recorded during ambulance trips to specify a common \(\nu_\ell\) for all \(\ell\). The constant \(\xi_u\) captures how strongly we believe our prior estimate \(\nu_\ell\) of the log unit travel time. We take \(\xi_u\) to be large, allowing the information in the data to dominate the posterior estimate of \(u_\ell\). Specifically, we set \(\xi_u\) so that there is roughly 95% prior probability that \(u_\ell\) is within a factor of two of \(e^{\nu_\ell}\), which corresponds to \(\xi_u = (\log 2)/2\). Similarly, \(\xi_\mu\) captures our prior uncertainty in the value of \(\mu_k\), and by the same argument we set \(\xi_\mu = (\log 2)/2\). We have no prior information about \(c\), \(M\), and \(\delta\), so we use uniform priors. Although these uniform prior distributions are non-integrable, the posterior distribution is integrable and valid. The uniform priors are on the square root of \(\delta\) and \(M\), because the square roots of these parameters are on the scale of the standard deviation of the log travel times, and it is more appropriate to put a uniform prior on a standard deviation than on a variance [8].

To estimate the posterior distribution for each parameter, we use a Metropolis-within-Gibbs Markov chain Monte Carlo method [27]. Specifically, we use Metropolis-Hastings (M-H) to update each of the unknown parameters in turn, conditional on the current values of the other unknown parameters. For example, to update the parameter \(u_\ell\), we propose a new value \(u_\ell^* \sim LN(\log(u_\ell), \psi^2)\). The proposed sample is accepted with the appropriate M-H acceptance probability, which is the minimum of 1 and the following product of the prior, likelihood, and proposal density ratios:

\[
\frac{\mathcal{L}(u_\ell^*; \nu_\ell, \xi^2_u) \mathcal{L}(u_\ell; \log(u_\ell^*), \psi^2) \prod_{i=1}^I \mathcal{L}(T_i; \mu_{k(i)} + \log(c + \sum_{j \in A_i} f_{ij} d_j u_{\ell(j)}^*), M e^{-\lambda d_i} + \delta)}{\mathcal{L}(u_\ell; \nu_\ell, \xi^2_u) \mathcal{L}(u_\ell; \log(u_\ell), \psi^2) \prod_{i=1}^I \mathcal{L}(T_i; \mu_{k(i)} + \log(c + \sum_{j \in A_i} f_{ij} d_j u_{\ell(j)}), M e^{-\lambda d_i} + \delta)}. 
\]
The variance $\psi^2$ is a constant that may be tuned to control the average acceptance probability, which theoretical evidence suggests should be roughly 23% for optimal efficiency [26].

To obtain the results in this article, we ran each Markov chain for 120,000 iterations, including a burn-in period of 20,000 iterations. To assess the Monte Carlo error, we calculated Monte Carlo standard errors for each of the parameter estimates, using batch means [16]. Standard errors are quite low, roughly 1-2% of the parameter estimate for the $\mu_k$ parameters and 0.03-0.2% for the other parameters. The computation time for each Markov chain iteration scales linearly with the number of vehicle trips, for a fixed road network. Each Markov chain run for these experiments takes roughly 18 hours on a personal computer, without utilizing parallel computing. Since the likelihood is a product over the terms for each trip, computation time could be decreased by calculating the likelihood terms in parallel batches. The Budge et al. nonparametric method [5] is estimated using maximum (penalized) likelihood [25] and is faster than our Bayesian implementation. In practice, however, ambulance travel time estimates are updated infrequently, so increased computation time is not a severe drawback [29].

3 Toronto EMS Data

We use our method for a study of ambulance travel times in Toronto, Ontario. The goal is to estimate the distribution of time required for an ambulance to drive to the scene of a high-priority emergency, in which case the ambulance uses lights and sirens, and travels at high speed. The data are provided by Toronto EMS (Emergency Medical Services), and include all such ambulance trips in Toronto during the years 2007 and 2008. We analyzed a subset of these data from the Leaside region of Toronto in previous work [29]; here we estimate travel times on the entire Toronto road network, which consists of 68,272 links.

The data associated with each trip include the approximate start and end times and locations of the trip, as well as sparse GPS location and speed readings during the trip. The GPS measurements are stored every 200 meters (m) of travel or 240 seconds (s), whichever comes first (typically the distance constraint is satisfied first for lights-and-sirens travel).

Cleaning and preprocessing the data is a challenge, due to factors such as human error in recording the start and end times and locations of the trips, the presence of trips where the ambulance doubled back on itself, and the presence of GPS measurement error. These challenges and our preprocessing algorithm are described in Appendix A. After preprocessing we are left with 157,283 ambulance trips, having removed 20,443 trips. The median shortest-path distance
between the start and end locations is 2,530 m.

To apply our method, we first estimate the path traveled for each ambulance trip, using the GPS data. Many such “map-matching” methods could be used [18, 19, 22, 23]; we use the one introduced in [28].

3.1 Exploratory Analysis

Here we highlight exploratory analysis of the Toronto EMS data, after trip preprocessing. Results from this analysis motivate the modeling assumptions described in Section 2.1 for the ambulance study. After preprocessing, each trip consists of a sequence of GPS readings. To assist exploratory analysis of the travel time distribution between any two locations, we map the first and last GPS readings of each trip to the nearest intersections in the network, to use as estimated start/end locations (this differs from our travel time model, in which trips are allowed to start and end in the interior of links). We collect the most common pairs of start/end intersections for the trips in the dataset; there are 10 start/end pairs with at least 40 trips between them.

Figure 1 shows normal Quantile-Quantile (Q-Q) plots for the log travel times between the four most common start/end pairs in the dataset. The shortest-path distance between the start and end locations is shown above each Q-Q plot. Also shown on the Q-Q plots are 95% pointwise confidence bands, under the null hypothesis that the log travel times are normally distributed. Only 6% of the observed travel times in the four plots fall outside the pointwise confidence bands, which suggests that the lognormal assumption is reasonable (if it is correct then we expect roughly 5% of the observations to fall outside of the bands). Although nearly all of the points outside the confidence bands occur on a single one of these four plots, this is not surprising because the points on a Q-Q plot are strongly dependent. Similar Q-Q plots can be constructed for the next most common start/end pairs, and they also suggest lognormal travel times.

The lognormal distribution has been observed and used as a model for both link and trip travel times repeatedly in the literature [11, 12, 15, 20]. We use the lognormal distribution because it is supported by exploratory data analysis, and also to provide a parsimonious parametric model. Due to the sparseness in data, there are typically few trips between any two locations in the road network. While Budge et al. found that ambulance travel times were heavier-tailed than the lognormal (they used log-t distributions), they did not condition on the start and end
Figure 1: Normal Quantile-Quantile plots for travel times between the four most common start/end location pairs in the Toronto EMS dataset.

locations of the trips. For the Toronto ambulance data, if one does not condition on the start and end locations, the travel time distribution also has heavier tails than the lognormal.

We also wish to investigate the variability in travel times for each start/end location pair. Figure 2 shows a scatterplot of the sample variances of the log trip travel times for the 100 most common start/end pairs, plotted against the shortest-path distance between the pair. There is a general decreasing trend in the variance, the shape of which suggests the exponential decay model described in Section 2.1. This is for the log travel times; on the original scale, the variances increase with distance. One can also construct a similar scatterplot where each point represents trips of a similar path distance, not just between specific locations. In this case, we again observe a decreasing trend, but with much less noise than in Figure 2. This is consistent
with the results seen by Budge et al., who observed decreasing coefficient of variation of travel times with increasing distance.

4 Application of TomTom

TomTom is a maker of navigation products. These products use both historical travel time averages and real-time traffic information observed by TomTom devices in vehicles to provide travel time estimates between any two points in the road network. These are estimates of average travel time, intended for use by standard vehicles. Such circumstances differ from our context; for instance, ambulances at lights and sirens (L-S) speed travel faster than standard vehicles. However, TomTom estimates provide a useful comparison. We report results using their historical average travel time estimates after adjusting for bias (see Section 5.1). Bias adjustment does not fully account for the differences between the TomTom context and ours; for example, intersection effects are much lower for L-S ambulances because they do not stop for red lights. So our results should not be interpreted as an evaluation of the quality of TomTom’s estimates. On the contrary, the fact that TomTom’s estimates are competitive with the other methods (see Section 5.1) shows that standard vehicle travel times can be useful for predicting L-S travel times.
5 Results

Here we give the results of ambulance travel time estimation using the Toronto data. We compare our proposed method, our previous method described in [29], the nonparametric method of Budge et al., and the TomTom predictions. For our proposed method, we use seven road classes and four time bins. Class 1 corresponds to highways, Class 2 to major arterial roads, Classes 3-6 to smaller-sized roads in decreasing order, and Class 7 to highway on and off-ramps. Time Bin 0, the baseline bin, corresponds to weekday off-peak times (10 a.m. - 3 p.m., 7-10 p.m.), Bin 1 to rush hour (6-10 a.m., 3-7 p.m.), Bin 2 to weekend daytime (6 a.m. - 10 p.m.), and Bin 3 to late night (10 p.m. - 6 a.m.). We chose these bins by observing the change in average GPS speed readings across the week.

We split the ambulance trips randomly into two equal-sized sets, using half of the data to train (estimate the parameters of) the statistical models, and the other half as test data for all the methods. Then we reverse the training and test halves. Results from these two experiments are similar. Table 1 gives parameter estimates from our model for the first training set.

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Table 1: Parameter estimates from our model, along with 95% intervals expressing parameter uncertainty.

The road class parameter estimates appear reasonable. The estimated unit travel time $u_1 = 0.0353$ s/m for Class 1 (highways) corresponds to approximately 102 km/hr. For Class 7 (highway on/off ramps), $u_7 = 0.0450$ s/m corresponds to approximately 80 km/hr, and for Class 2 (major arterial roads), $u_2 = 0.0603$ s/m corresponds to approximately 60 km/hr. The
estimated speeds decrease for smaller roads, except for Class 6, the smallest roads. These roads are relatively uncommon, and the interval estimate is wider for \( u_6 \) than for the other parameters, reflecting larger uncertainty in the value of \( u_6 \).

The rush hour time bin parameter estimate \( \mu_1 = 0.0268 \) corresponds to a travel time increase of about 2.7% for rush hour, relative to weekday off-peak. The estimates of \( \mu_2 \) and \( \mu_3 \) correspond to roughly 1% smaller travel times for weekend and late night, relative to weekday off-peak. All these values are close to zero, indicating that lights-and-sirens ambulance speeds are remarkably consistent across time bins, in contrast to standard travel speeds (see Section 5.3 and [29]).

Our lognormal model implies that about 95% of trips are predicted to fall within two standard deviations of the median on the log scale, i.e. within factors of \( e^{-2 \times \text{SD}} \) and \( e^{2 \times \text{SD}} \) of the median on the original scale. So the variance estimate \( \delta = 0.0576 \) means that for very long trips about 95% of the travel times will be within factors of 0.62 and 1.6 of their median travel time. The estimate \( M = 0.2064 \) implies that for very short trips about 95% of the travel times will be within factors of 0.36 and 2.8 of their median travel time.

### 5.1 Travel Time Prediction Comparison

Next we compare the predictive performance for our method, several reduced versions of our method, the nonparametric method of Budge et al. [5], and the TomTom estimates. Recalling that we use half of the data for training and the other half for testing and then reverse, here we evaluate the accuracy of the predicted travel time distribution for trips in the test data. For each test trip we evaluate the quality of a point estimate of the travel time, the predictive interval estimate, and the distribution estimate using appropriate statistical measures. For TomTom we only evaluate the quality of the point estimate since interval and distribution estimates are not available. For our method and that of Budge et al., we use the median travel time as the point estimate of travel time. The 95% predictive interval from those methods is taken to be the estimated 0.025 and 0.975 quantiles of the travel time distribution.

When using our method to predict the travel time for the trips in the test data, we obtain predictions under two scenarios: (1) using the estimated route taken by the vehicle (based on the GPS data), or (2) not using this information. Using the estimated route emulates a situation in which we know the route that the driver will take, for instance if the driver were required to take a route specified by the dispatcher. Such control over the route could be desirable since then the route could be optimally selected using the most recent traffic conditions. However, most
ambulance organizations leave the route choice to the driver, without notifying the dispatcher of their choice. To emulate this situation, in Scenario 2 we predict the travel time without using the route information (only using the start and end locations of the trip). In this scenario we obtain an estimated fastest route according to our model (as described in Appendix B), and base our predictions on this route.

Budge et al. base their travel time predictions on the shortest-path distance between the start and end locations [5]. In the spirit of Scenario 1, since we have estimated routes for each ambulance trip, it is natural to extend their method to use the distance of the estimated route, instead of the shortest-path distance. Therefore, we obtain predictions from their original method where the training and test sets both use the shortest-path distance, and the extended method where the training and test sets both use the estimated route.

We perform bias correction for each estimation method, since bias may be present for a variety of reasons. For example, bias arises in Scenario 2 introduced above because in this scenario our method treats the ambulance paths differently in the training and test data. For the training trips the estimated route is used, while for the test trips the fastest route is used, resulting in a tendency to underestimate travel times. Bias may also be present in each method due to inaccuracies of the assumed model. The TomTom estimates are severely biased, because they are intended for vehicles traveling at standard speeds, not lights-and-sirens speeds. We do bias correction on the log scale via cross-validation as described in previous work [29]. Bias correction is done on the log scale to lessen the impact of outlying travel times.

Results are shown in Table 2. We report point estimation performance using the root mean squared error (RMSE, in seconds) of the point estimate compared to the true time, and using the RMSE of the log predictions compared to the true log time (“RMSE log”). Due to the inherent variability in travel times, even a perfect distribution estimate would have RMSE and RMSE log considerably above zero. We report the RMSE log because it is less affected by outlying travel times than the RMSE. Outliers are present for at least two reasons; first, a small number of trips were not driven at typical lights-and-sirens speeds, although they were recorded as high-priority trips. Second, some trips have high error in the recorded GPS locations, in which case the estimated path may be inaccurate.

To evaluate the interval estimates, Table 2 shows the percentage of test trips where the observed travel time falls in the 95% predictive interval (the coverage, “Cov. %”), as well as the geometric mean width of the 95% predictive intervals (“Width”). Coverage close to or above 95% combined with small interval width is desirable, since it indicates that the predictive
<table>
<thead>
<tr>
<th>Estimation method</th>
<th>RMSE (s)</th>
<th>RMSE log</th>
<th>Cov. %</th>
<th>Width (s)</th>
<th>CRPS (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Our method, using estimated route</td>
<td>72.3</td>
<td>0.298</td>
<td>94.4</td>
<td>218.9</td>
<td>34.6</td>
</tr>
<tr>
<td>Our method, using fastest route</td>
<td>77.7</td>
<td>0.322</td>
<td>93.1</td>
<td>219.7</td>
<td>37.3</td>
</tr>
<tr>
<td>Our method, 1 variability parameter</td>
<td>72.5</td>
<td>0.297</td>
<td>94.1</td>
<td>225.9</td>
<td>35.2</td>
</tr>
<tr>
<td>Our method, 1 time bin</td>
<td>72.4</td>
<td>0.298</td>
<td>94.4</td>
<td>219.1</td>
<td>34.7</td>
</tr>
<tr>
<td>Our method, 1 road class</td>
<td>76.8</td>
<td>0.312</td>
<td>94.3</td>
<td>231.0</td>
<td>36.7</td>
</tr>
<tr>
<td>Extended Budge et al.</td>
<td>74.9</td>
<td>0.302</td>
<td>94.6</td>
<td>229.1</td>
<td>35.7</td>
</tr>
<tr>
<td>Budge et al.</td>
<td>79.7</td>
<td>0.325</td>
<td>94.8</td>
<td>248.1</td>
<td>38.3</td>
</tr>
<tr>
<td>TomTom</td>
<td>82.1</td>
<td>0.347</td>
<td>NA</td>
<td>NA</td>
<td>NA</td>
</tr>
</tbody>
</table>

Table 2: Travel time prediction performance for the Toronto EMS lights-and-sirens data.

distribution is narrowly concentrated around the true travel time, while reflecting the true variability in travel times.

Table 2 evaluates the quality of the distribution estimates by reporting the continuous ranked probability score (CRPS) [9]. This is a “strictly proper” measure of distribution estimation accuracy, meaning that only a perfect distribution estimate achieves the lowest expected score [10]. If $F$ is the estimated distribution function and $x$ is the observed travel time, $\text{CRPS}(F; x) \triangleq \int_{-\infty}^{\infty} [F(y) - 1(y \geq x)]^2 \, dy$ is the integrated square of the difference between $F$ and the empirical distribution function based on the single observation $x$ [9]. We report the mean CRPS over the test trips [9]; a lower value corresponds to better distribution estimates. Even a perfect distribution estimate would yield a CRPS value well above zero, due to the inherent variability of travel times.

In Table 2 in addition to reporting the accuracy of our method under Scenarios 1 and 2, and the accuracy of the competing methods, we report the accuracy of several simplified versions of our method under Scenario 1. This indicates whether the simplified models are as effective as our full model and which aspects of our full model are the most important. We consider the following simplifications: (a) only one time bin, (b) only one road class, and (c) only one variability parameter instead of the exponential model.

As seen in Table 2 our method under Scenario 1 (using the estimated route) outperforms the Budge et al. method by 8-10% in RMSE, RMSE log, and CRPS, and outperforms the extended Budge et al. method by 1.5-3.5% in the same metrics. Our method’s interval estimates have
almost identical coverage to those of Budge et al. but are narrower on average, by 12% compared
to the original Budge et al. method and by 4.5% compared to the extended method. Under
Scenario 2, our method outperforms the original Budge et al. method by 2.6% in CRPS and
1-3% in RMSE and RMSE log. Our mean predictive interval width under this scenario is 11%
narrower than that of Budge et al., though with slightly lower coverage. These performance
differences are most likely due to our model’s inclusion of different speeds for the different road
classes, as well as time effects.

Our method outperforms the TomTom estimates by 12-14% in RMSE and RMSE log under
Scenario 1, and by 5-7% in the same metrics under Scenario 2. Scenario 2 is the more natural
comparison, because we do not specify the route traveled when obtaining the TomTom estimates,
instead allowing TomTom to pick the optimal route. TomTom’s estimates perform respectably,
indicating that after bias correction, standard vehicle data do have predictive power for lights-
and-sirens ambulance trips.

Regarding the reduced versions of our approach, the method with only one time bin performs
essentially as well in all metrics as the full method. We explore this observation in more detail in
Section 5.3. The model with only one variability parameter performs as well in point estimation
but slightly worse in distribution estimation than the full model. The method with only one
road class performs worse than the full method and the other reduced methods in all metrics.
Therefore, it is quite important to allow for varying speeds across road classes (also discussed
in previous work [29]). The extended Budge et al. method outperforms our method with one
road class. Both models rely only on travel distance; however, the Budge et al. method is more
flexible than our method with one road class, because the point estimates on the log scale are
not restricted to a linear function of distance.

5.2 Comparison to our Previous Method

We also wish to compare to our earlier method as described in [29], referred to as Westgate et al.
Our previous method is much more computationally intensive than the method proposed here
because it simultaneously estimates the routes of the historical trips and travel time parameters
of each network link. Because of this, we cannot apply it to the entire Toronto road network,
so we compare our new method to our previous method on the subregion of Leaside, Toronto.
To ensure a fair comparison, we do not use the route information for the test trips with either
method (i.e., we use Scenario 2 from Section 5.1).
For application to the subregion we make one minor change to the model introduced in Section 2.1. For the prior distribution on the variance parameter $M$, we use an exponential distribution with rate 5, instead of a uniform distribution. Since the dataset has few extremely short trips, posterior estimates of $M$ are unstable unless we use a prior distribution that prefers smaller values. Failure to do this can lead to unrealistic travel time predictions for the few extremely short trips in the dataset.

Results are summarized in Table 3. We use the same five resamplings of training and test sets from the Toronto subregion data as in [29]. The two methods perform roughly the same in terms of RMSE log, and our previous method performs only slightly better than our new method in RMSE, even though the new method is much less computationally intensive. Our new method also has much better coverage of interval estimates than our previous method. This is because our previous method assumed independence between the travel times on different network links, which is unrealistic, as discussed in Section 1. Failing to take into account the association between link travel times leads to underestimation of the variability in the total route travel time and thus overly narrow interval estimates.

<table>
<thead>
<tr>
<th>Estimation method</th>
<th>RMSE (s)</th>
<th>RMSE log</th>
<th>Cov. %</th>
<th>Width (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Westgate et al.</td>
<td>37.8</td>
<td>0.332</td>
<td>85.8</td>
<td>75.0</td>
</tr>
<tr>
<td>Our method, using fastest route</td>
<td>38.1</td>
<td>0.331</td>
<td>91.3</td>
<td>90.3</td>
</tr>
</tbody>
</table>

Table 3: Travel time prediction performance of our proposed method and previous method on the subregion of Leaside, Toronto.

### 5.3 Inflation of Time Effects

In Section 5.1, we observed that the inclusion of time effects did not noticeably improve performance of our method on the Toronto EMS data. For ambulance fleets in other municipalities and for non-ambulance contexts, the differences in travel times across time bins may be greater. For instance, although the difference in travel speeds between rush hour and non-rush hour on the full Toronto dataset is only about 4% (obtained by comparing GPS speed readings), this difference is 8% if one restricts to the Leaside subregion of Toronto, and is 16% for standard speed (not “lights-and-sirens”) ambulance data on the Leaside subregion. In this section, we artificially inflate the travel times for trips in the rush hour time bin, to see what effect the
inclusion of time bin factors has on performance if the differences across time bins are larger.

We multiply each trip travel time in the rush hour time bin by an artificial inflation factor and apply our model using both one time bin and four time bins. The inflation percentages used are 5% (inflation factor 1.05), 10%, and 20%. The estimated routes from the GPS data are used for the test trips (Scenario 1 in Section 5.1). Results are given in Table 4. For small rush hour inflation, the difference between the 4 time bin model and the 1 time bin model is minimal. However, the difference increases in a nonlinear manner with the increasing inflation, and becomes fairly large (6%) at 20% inflation. We expect that our method with multiple time bins would show substantial improvement over the method with one time bin on a dataset where the travel time difference between rush hour and non-rush hour is 20% or more.

<table>
<thead>
<tr>
<th>Rush hour inflation percentage</th>
<th>No inflation</th>
<th>5%</th>
<th>10%</th>
<th>20%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Our method, 4 time bins</td>
<td>72.3</td>
<td>73.2</td>
<td>74.2</td>
<td>76.3</td>
</tr>
<tr>
<td>Our method, 1 time bin</td>
<td>72.4</td>
<td>73.9</td>
<td>75.9</td>
<td>81.0</td>
</tr>
</tbody>
</table>

Table 4: Travel time prediction performance (RMSE), with rush hour travel time inflation.

5.4 Closest Ambulance Post Comparison

In this section and the next, we consider the effect of using different travel time distribution estimates on ambulance fleet management. First, we assume a set of locations of available ambulances, and calculate which ambulance is estimated to be closest in terms of travel time to each intersection in the city, according to our method and the Budge et al. method. If the two methods estimate different ambulances to be closest to a particular intersection, this would lead an ambulance dispatcher to assign different ambulances to respond to an emergency at that intersection, if the policy is to dispatch the closest ambulance.

We select a set of twenty-five representative ambulance post locations in Toronto, by examining the empirical distribution of start locations of ambulance trips (after data preprocessing), and choosing commonly-occurring locations. These ambulance posts are chosen to illustrate and compare the travel time estimates from our method and the Budge et al. method, and are not indicative of actual ambulance coverage of Toronto EMS.

For our method, we define the closest post to an intersection to be the one with the smallest
estimated median travel time. This corresponds to Scenario 2 from Section 5.1. For the Budge et al. method, we use the closest post in shortest-path distance. Typically this coincides with the closest post according to median travel time, since their method models median travel time as a function of only shortest-path distance. However, it is not guaranteed since Budge et al. do not restrict the function to be increasing in distance, and for the Toronto data the estimated function does have small non-monotonic fluctuations.

Figure 3: Intersections (shown in black) where the closest ambulance post differs when estimated by our method and the Budge et al. method, with post locations shown as X’s.

In Figure 3, black points mark the intersections that are estimated to be closest to different posts, according to our method and the Budge et al. method. Light gray points represent the remaining intersections. The ambulance post locations are shown as black X’s. Roughly 5% of the intersections in the city are estimated to be closest to different posts. Typically, only
intersections that are roughly halfway between two posts have a chance to be marked. Therefore, if the number of ambulance posts were higher, it is likely we would see even more intersections marked. By comparison, roughly 10% of the intersections in the city are closest to different posts according to straight-line distance and shortest-path distance. Therefore, the 5% we observe comparing our method and the Budge et al. method is fairly large.

5.5 Probability of Arrival Within a Time Threshold

In this section, we calculate the probability that an ambulance is able to reach each intersection in the city within a time threshold, given a set of currently available ambulance locations and a travel time distribution estimate for any path. Visual displays of these probabilities are called probability-of-coverage maps [5, 29]. We use the same set of twenty-five representative ambulance posts and the same methods for estimating the closest post to each intersection as in the previous section.

In Figure 4, we plot the probability that an ambulance arrives at each intersection in Toronto from the closest ambulance post within 4 minutes, according to our method. Each intersection is shaded in gray according to this probability, where darker points correspond to higher probability. The post locations are shown as white X’s. The probability of arrival is very high for intersections near the closest post and becomes lower for intersections farther away.

The arrival probabilities from our method do not decrease solely as a function of travel distance from the closest post, but also incorporate road speeds. This becomes clear in the top panel of Figure 5, where we plot the differences between the arrival probabilities for our method and the Budge et al. method. The black points represent intersections where our method gives at least 15% higher probability of arrival within 4 minutes than the Budge et al. method does. Thus, there is a substantial predictive difference between the two distributions for these intersections. The medium grey points are intersections where the Budge et al. method gives at least 15% higher probability than our method does. The light gray points are all other intersections. The ambulance post locations are again shown as black X’s.

Most of the intersections that are close to an ambulance post do not differ by 15% or more according to the two methods, because arrival probabilities from both methods are high. Similarly, intersections that are far from all ambulance posts also differ by less than 15%. On the other hand, many of the intersections that are at an intermediate distance to the closest ambulance post differ in arrival probability by 15% or more. In fact, this is true for roughly 10%
Figure 4: Probability of arriving at each intersection in Toronto from the closest ambulance post within 4 minutes, estimated by our method.

of all the intersections in the city. Many of the points where the probability from our method is at least 15% higher are on or near highways, particularly Highway 401, which is visible in Figure 5 as a sequence of black points running horizontally across the middle of the city. The highway road class speed estimate is high, so the method predicts better coverage when a highway can be used. There is another large collection of black points at the left edge of the figure that are close to Highway 427.

Many of the intersections where the Budge et al. probability is at least 15% higher are in residential areas where there is no direct path following highways or major arterial roads. For example, there are no major roads traveling from an ambulance post to the collection of gray
Figure 5: Differences in the estimated probability of arriving within 4 minutes, between our method and that of Budge et al.

points near location (-10000, -7000). Similarly, there is no direct route from an ambulance post to the collection of gray points near location (-5000, 7000). Though there are major arterial roads in the area, it would require a detour to use one. There are smaller roads that take more direct routes from the ambulance posts, but these road classes have slower speed estimates.

6 Conclusions

We introduced a parametric model for estimating the distribution of ambulance travel times between any locations in a road network. The method uses data from historical ambulance trips that can be sparse in time and network coverage, and is computationally tractable for large
road networks and large datasets of vehicle trips. The model parameters are interpretable, and include effects for the roads traveled by the vehicle and trip-level effects such as time of day. We used a Bayesian formulation and Markov chain Monte Carlo method to estimate the model parameters.

We tested the method on a large dataset of ambulance trips from Toronto. Exploratory analysis of the data indicated that the distribution of ambulance travel times between two fixed locations is well modeled by a lognormal distribution, with variability parameter depending on travel distance. These observations influenced our modeling choices. We compared travel time predictions from our method with predictions from a recently-published method by Budge et al. [5] and commercially-available travel time estimates from TomTom. We found that our method outperformed the alternative methods in both point estimation and distribution estimation. We also compared our method with the method of Westgate et al. [29] on a subregion of Toronto, and found that our method performed almost as well in point estimation and better in interval estimation, while being far more computationally efficient.

We also investigated several reduced versions of our method, to determine which features were the most important. The largest benefit came from the inclusion of parameters for each road class in the city, compared to a model with only one road class. However, there was little benefit in performance from adding multiple time bins across the week vs. a single time bin. In the Toronto dataset, the ambulance travel times do not vary substantially across the day and week, even during rush hour. Because other cities or datasets may be more variable in time, we performed an additional set of experiments by artificially inflating the difference in travel times between time bins. We found that if the travel times during rush hour were increased by at least 20%, then time bin factors provided a substantial benefit to estimation.

Finally, we investigated operational differences for ambulance fleet management from using our method vs. the method of Budge et al. After fixing a set of representative ambulance posts in Toronto, we calculated the closest post to each intersection in the city, according to travel time estimates from the two methods. We found that the two methods estimated 5% of the intersections in the city to be closest to different posts, which could lead to different dispatch decisions for emergencies occurring at these intersections. We also calculated the probability that an ambulance arrives at each intersection in the city within 4 minutes, responding from the closest post. We found that for about 10% of the intersections in the city, the two methods gave arrival probabilities that differed by more than 15%.
A Preprocessing

For each ambulance trip we have the time the ambulance departed for the scene (enroute time), the arrival time, and GPS readings for the ambulance between those two times. Ideally, we would use the difference between the enroute and arrival times as the total trip travel time, and use the GPS readings between the enroute and arrival times to estimate the path traveled via a map-matching algorithm. However, the enroute and arrival times are error-prone. They are manually recorded inside the ambulance by a button push, and sometimes the button is pushed at the wrong time. For example, sometimes the button indicating arrival at the scene is not pushed until after the ambulance *departs* from the scene. The GPS device continues to record data, so there will be many consecutive readings with speed 0 in between the recorded enroute and arrival times, while the ambulance is parked at the scene. A stylized example of this issue is given in Figure 6.

![Figure 6: A stylized example of the effect of error in recorded enroute and arrived times.](image)

Instead of using these error-prone enroute and arrival times, we estimate the start and end locations and times using the GPS data. First, to extract only the GPS readings where the ambulance was actually driving to the scene, we isolate the first “traveling block” (defined below) of GPS points, and discard the rest. Then we take the first and last GPS points of the traveling block as the estimated start and end locations and times of the trip. Due to GPS measurement error, these locations are not necessarily on the road network, but the map-matching algorithm we use can handle this discrepancy [28].

A traveling block is a maximal consecutive sequence of GPS readings, with the requirements:

1. For each incident with priority Echo or Delta, extract all GPS points with timestamps between the recorded enroute and arrived times.

2. For each trip, retain the first “traveling block” of GPS points, discarding the rest.

**Traveling block:** A maximal consecutive sequence of GPS readings, with the requirements:
1. Begins and ends with a non-zero GPS speed.

2. Has at least 3 non-zero speed GPS readings.

3. Has no pair of GPS readings (consecutive or otherwise) with:
   
   (a) Timestamps at least 30 seconds apart but with average speed < 0.5 m/s, using straight-line distance.
   
   (b) Timestamps at least 2 minutes apart but with average speed < 2 m/s, using straight-line distance.
   
   (c) Average speed (straight-line) greater than 100 m/s.

4. Has straight-line distance of at least 400 m between the first and last GPS readings.

5. Has average speed (based on straight-line distance) between the first and last GPS readings no greater than 60 m/s.

Each of these requirements are designed to eliminate a certain type of error. Requirement 1 removes zero-speed GPS readings at the beginning or end of the trip. Requirement 2 ensures that we can estimate start and end locations for the trip, with at least one additional GPS reading for path estimation. Requirement 3 ensures that the trip does not have a long stationary period in the middle, as in Figure 6. This requirement also removes trips where the ambulance turned around, and subsequent GPS readings are very close to each other. While this is possible behavior, it is unhelpful for response time estimation to include these trips. Finally, this requirement also removes trips with severe errors in the GPS timestamp or location. Errors in the GPS data are not common, but occasionally the data contain successive GPS readings with identical timestamps but different locations, or GPS readings with impossible locations. Requirements 4 and 5 act similarly to Requirement 3, but on the entire trip. Requirement 4 removes trips where the ambulance turned around and the first and last GPS reading are very close to each other. Requirement 5 removes rare trips where the GPS data are shifted by a very large amount from the true location.

### B Fastest Path Estimation

Here we describe the fastest path estimation for our method under Scenario 2 of Section 5.1. As noted in Appendix A, the recorded start and end times for the ambulance trips are error-prone, so the first and last GPS readings in the first traveling block of the trip are used for the start and
end times and locations. Since these two locations are not necessarily on the road network, to estimate the fastest path we first find the two nearest links to these GPS locations, and use the nearest points on these links as possible start/end locations. These links typically correspond to the two travel directions of the nearest road. For each of the four start/end location pairs, we calculate the fastest path in median travel time. Of these four possible paths, we use the one with the smallest median travel time as the estimated path. This method ensures that we obtain a reasonable path for each trip, which can begin or end in the interior of a link, and is not hampered by choosing the “wrong direction” of the nearest link.

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References


