Recitation 7B

1 Problems

1. A government space agency is conducting research project on an engineering problem that must be solved before people can safely fly to Mars. Three independent research teams are currently trying three different approaches to solve this problem.

   The probability that team 1 will fail to solve this problem is 0.40, the probability that team 2 will fail is 0.60, and the probability that team 3 will fail is 0.80. We say that the project fails if all three teams fail. So, currently, the probability that the project fails is $0.40 \cdot 0.60 \cdot 0.80 = 0.192$.

   The space agency decides to assign a total of three new scientists to the project. The three new scientists can be assigned to together or separately to any of the teams. The following table gives the new probabilities of failure if 0, 1, or 2 new scientist is assigned to each team. Our task is to use a dynamic programming approach to decide how many new scientists to assign to each team such that the probability of project failure is minimized.

Number of new scientists	Probability of failure		
	Team 1	Team 2	Team 3
0	0.40	0.60	0.80
1	0.20	0.40	0.50
2	0.15	0.20	0.30
3	0.10	0.17	0.25

   (a) Formulate a dynamic programming approach for this problem

   (b) Solve the problem by hand.

2. Suppose that we have a knapsack of weight capacity $W = 13$ and volume capacity $V = 15$. There are 4 different products with the following weight, volume, and value information.

   There is an infinite number of copies of each product available. Our objective is to decide how many copies of each product we should take in order to maximize the total value, subject to both weight capacity and volume capacitiy constraints.

Product (i)	Weight per unit (w_i)	Volume per unit (v_i)	Value per unit (d_i)
1	5	4	9
2	3	5	4
3	2	3	3
4	1	1	0.5

   (a) Formulate a dynamic programming approach for this problem

   (b) Solve the problem by hand or using AMPL.

2 Puzzles with DP

1. Suppose we want to make change for n cents, using the least number of coins of denominations 1, 10, and 25 cents. Formulate a dynamic programming approach for this problem. Solve the problem for $n = 68$.

2. A *contiguous subsequence* of a list S is a subsequence that is made up of consecutive elements of S. For instance, if S is the sequence

$$5, 15, -30, 10, -5, 40, 10,$$

then $15, -30, 10$ is a contiguous subsequence but $5, 15, 40 is not$. Formulate a dynamic programming approach for finding a contiguous subsequence of list S that has maximum sum. (Let a subsequence of length zero have sum zero.)

Hint: Suppose n = length of sequence S. For each $j \in \{1, 2, \ldots, n\}$, consider contiguouss subsequences of S ending exactly at position j.