Recitation 4

In this recitation, you will be using the max-flow model to solve the so-called baseball elimination problem, which your TA will introduce.

You can work in pairs or individually. Answer the questions in the blank space provided, and turn this handout in to your section TA. If you work with a partner, you and your partner need only turn in one copy.

Name and NetID:
Section:

1 The Baseball Elimination Problem

The input for the "Baseball Elimination Problem" consists of a set of n teams, namely teams $1, 2, \ldots, n$; the number $g(i, j)$ for each pair of teams i, j, which denote the number of games that remain to be played between them; and the number $w(i)$ for each team i, which denote the total number of games that has been won by team i so far. We assume that each team plays the same total number of games in the entire season.

Our objective is to determine if team n has been eliminated already. That is, even if team n wins each of its remaining games (against any of the other teams), there will always be some team with more wins than team n at the end of the season.

2 Example

Consider the following data for a 4-team league.

Team	Wins
1	8
2	10
3	10
4	1

Games Remaining:

vs.	1	2	3	4
1	-	3	3	6
2	3	-	6	3
3	3	6	-	3
4	6	3	3	-

1. Our favorite team, team 4, did not do very well so far. We would like to determine whether it still has a chance to finish in first place at the end of the season (where a tie for the first place is good enough too). How many games can team 4 possibly win during the season? Call this number W.

$W =$

2. Team 4 finishes in first place if no other team wins more games than W. What if team 1 already has 14 wins? Can our team come out first?

3. Team 1 has already won 8 games. At most how many games is this team allowed to win if we want to make sure that our team comes out first? What about teams 2 and 3? How can you express this amount in general, in terms of W and $w(i)$ for each team i?

4. We now want to formulate the problem of deciding if team n has been eliminated as a maximum flow problem. As described, construct a directed graph in which there is a source s and a sink t, and one node for each pair of teams $\{i, j\}$, where $i \neq j$ and both $i, j \in \{1, \ldots, n-1\}$, and one node for each team i, $i = 1, \ldots, n-1$; there is an arc from s to each pair of nodes $\{i, j\}$ of capacity equal to the number of games remaining between i and j, namely $g(i, j)$; there is an arc from "team node" i to t of capacity $W - w(i)$; finally, for each pair of nodes $\{i, j\}$, there are arcs to each of the nodes i and j of essentially infinite capacity (but making this capacity equal to the total number of remaining games will also be sufficiently large).

   Sketch the graph above for our example.

5. Without using any algorithm (that is, just by sight and intuition), construct a feasible flow in this graph. Any feasible solution can be interpreted as some of the remaining games having been played. For your feasible solution, which games have been played?

   Who won those games?

   What does the total flow that is passing through the pair node $\{1, 2\}$ correspond to in terms of what has happened in these games?

   What does the total flow passing through team node 3 correspond to, again in terms of what has happened in these games that have been played?

6. Use your AMPL model to compute the value of the maximum flow for this input (that is, construct the correct .dat file for your input and use maxflow.dat to solve for a maximum flow). You do not have to include your .dat file in your recitation exercise submission, but state your optimal solution and its corresponding objective value.

7. What does this say about Team 4's chances to end the season in first place? If Team 4 has not been eliminated, then give a scenario by which team 4 could end up the season in at least a tie for first place. If Team 4 has been eliminated, give a short explanation why team 4 has been eliminated (without refering to the maximum flow or minimum cut problem in any way).

8. If team 4 was not eliminated, then how many games from the restof the season could it lose, and still come in first place? Does it matter which of its remaining games it loses?

9. If team 4 was eliminated, how many additional games should it have won from the first part of the season in order to have prevented this early end to its competitive season? Could these additional wins come against any team for team 4 to remain "alive"?

10. In fact, although it is not relevant to understanding this type of formulation, the data that this example was based on is flawed. Can you explain why this could not have been real data? (Hint: how many games are played in total, throughout the league, for the entire season?)

11. Now, let's turn to some (almost) real data. On Friday August 30, 1996, the American League East standings were:

Team	Wins	Losses
New York	75	59
Baltimore	71	63
Boston	69	66
Toronto	63	72
Detroit	49	86

Each player plays a 162-game season, but some of the remaining games are played against teams outside of their division. Things look pretty bleak for Detroit, but are they eliminated? Within the division, here is the breakdown for games among the teams other than Detroit.

Team	Baltimore	Boston	New York	Toronto
Baltimore	-	2	3	7
Boston	2	-	8	0
New York	3	8	-	7
Toronto	7	0	7	-

This data must be sifficient to determine whether Detroit is eliminated. Why is all of the remaining data is irrelevant? (This is a simple answer; you don't need to do any calculations to determine the answer.)

12. Set up a maximum flow input that will determine the answer, and then use AMPL to solve it. Is Detroit eliminated? Would Detroit still have been eliminated if they had won just one more game earlier in the season?