Recitation 8

In this recitation, we will look at another stochastic dynamic programming problem.

Name and NetID:
Section:

1 Problem

(From Hillier and Lieberman, Chapter 10, Example 6)

A manufacturing company has received an order to supply one item of a particular type. However, the customer has specified such stringent quality requirements that the manufacturer may have to produce more than one item to obtain an item that is acceptable. The number of *extra* items produced in a production run is called the *reject allowance*. Including a reject allowance is a common practice when producing for a custom order, and it seems advisable in this case.

The manufacturer estimates that each item of this type that is produced will be *acceptable* with probability 0.5, and *defective* (without possibility of rework) with probability 0.5. Thus, the number of acceptable items produced in a lot of size L will have a binomial distribution. That is, the probability of producing no acceptable items in such lot is $(0.5)^L$.

The production cost for this product is \$100 per item (even if the item turns out to be defective), and excess items are worthless.

In addition, a setup cost of \$300 must be incurred whenever the production process is set up for this product, and a completely new setup at this same cost is required for each subsequent production run if a lengthy inpsection procedure reveals that a completed lot has not yielded an acceptable item.

The manufacturer has time to make no more than three production runs. Additionally, at each run, suppose that the manufacturer can make at most 5 units of products.

If an acceptable item has not been obtained by the end of the third production run, the cost to the manufacturer in lost sales income and penalty costs will be \$1600.

The objective is to determine the policy regarding the lot size (1 + reject allowance) for the required production run(s) that minimizes total expected cost for the manufacturer.

2 A dynamic programming formulation

1. Stage n corresponds to production run n (where $n = 1, 2, 3$).
2. State S_n = the number of acceptable items remain to be fulfilled at the beginning of stage n.

   So, $S_n = 1$ if we still need to produce an acceptable item at the beginning of stage n, and $S_n = 0$ if we no longer need to produce an acceptable item at the beginning of stage n (for example, if the production run at stage $n-1$ already succeeds in producing an acceptable item).

3. Decisions to make at stage n

4. Word-description of the optimization function $f_n^*(s)$

5. Boundary conditions

6. Recurrence relation

7. Computation

Summary (DP Table):

State	Stage 1	Stage 2	Stage 3
0			
1			

8. What is the optimal solution to the problem?