Recitation 13

In this recitation, we will work on solving several unconstrained and constrained nonlinear programming problems.

Name and NetID:
Section:

1 Newton's Method for Solving Unconstrained NLP

1. Consider the following problem:

$$\min_{x \in \mathbb{R}} \quad f(x)$$

where $f(x) = 3x^4 - 2x^3 + 4x^2 - 10x - 1$.

(a) Check if f is a convex function.

(b) Write down the optimality conditions for the above problem.

(c) Try solving for a point $\bar{x}$ that satisfy the optimality condition in (b) directly.

(d) Using $x^{(0)} = -1$ as the starting point, carry out 6 iterations of Newton's method to find a point that satisfy the optimality condition in (b). Call the solution from your fifth iteration $\tilde{x}$.

(e) Check if $\tilde{x}$ satisfy the optimality conditions that you found in (b).

2. Consider the following problem:

$$\min_{x \in \mathbb{R}^3} \quad f(x)$$

where $f(x) = x_1^4 + 1.5x_1^2 + 2x_1x_2 + x_1x_3 + 4x_2^2 + 2x_3^2 + 4x_1 - x_2 + 2x_3 - 5$.

(a) Check if f is a convex function.

(b) Write down the optimality conditions for the above problem.

(c) Try solving for a point $\bar{x}$ that satisfy the optimality condition in (b) directly.

(d) Using $x^{(0)} = (0, 0, 0)$ as the starting point, carry out 4 iterations of Newton's method to find a point that satisfy the optimality condition in (b). Call the solution from your fifth iteration $\tilde{x}$.

(e) Check if $\tilde{x}$ satisfy the optimality conditions that you found in (b).

2 Constrained NLP

1. Consider the following problem:

$$\begin{array}{rll} \min\limits_{x \in \mathbb{R}^3} & f(x) & \\ s.t. & g_1(x) & \geq 0 \\ & g_2(x) & \geq 0 \\ & h_1(x) & = 0 \end{array}$$

where

$$\begin{array}{rcl} f(x) & = & x_1 + x_3 + x_2^2 \\ g_1(x) & = & x_1 \\ g_2(x) & = & -x_1^2 - x_2^2 - x_3^2 + 16 \\ h_1(x) & = & x_1 + x_2 + x_3 - 4 \end{array}$$

(a) Check if the Lagrangian, $L_{y,z}(x)$, is a convex function.

(b) Write down the optimality conditions for the above problem.

(c) Solve for a point $\bar{x}$ that satisfy the optimality condition in (b).

(d) Using $x^{(0)} = (0, 0, 4)$ as the starting point, carry out 4 iterations of Newton's method to find a point $\tilde{x}$ that satisfy the optimality condition in (b).