Lecture 3

Previously in Opt 2 …

The minimum-cost flow problem

- Input to min-cost flow:
  - G = (N, E), a directed graph
  - b_i = node supply values, where $\sum_{i \in N} b_i = 0$
  - c_{ij} = edge costs
  - u_{ij} = edge capacities, $u_{ij} \geq 0$
- Objective: To minimize total cost, subject to constraints:
  - Capacity constraints
  - Flow conservation constraints:

Flow out of node i – Flow into node i = b_i

The minimum-cost flow problem

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Net flow out of node i = b_i

The minimum-cost flow problem

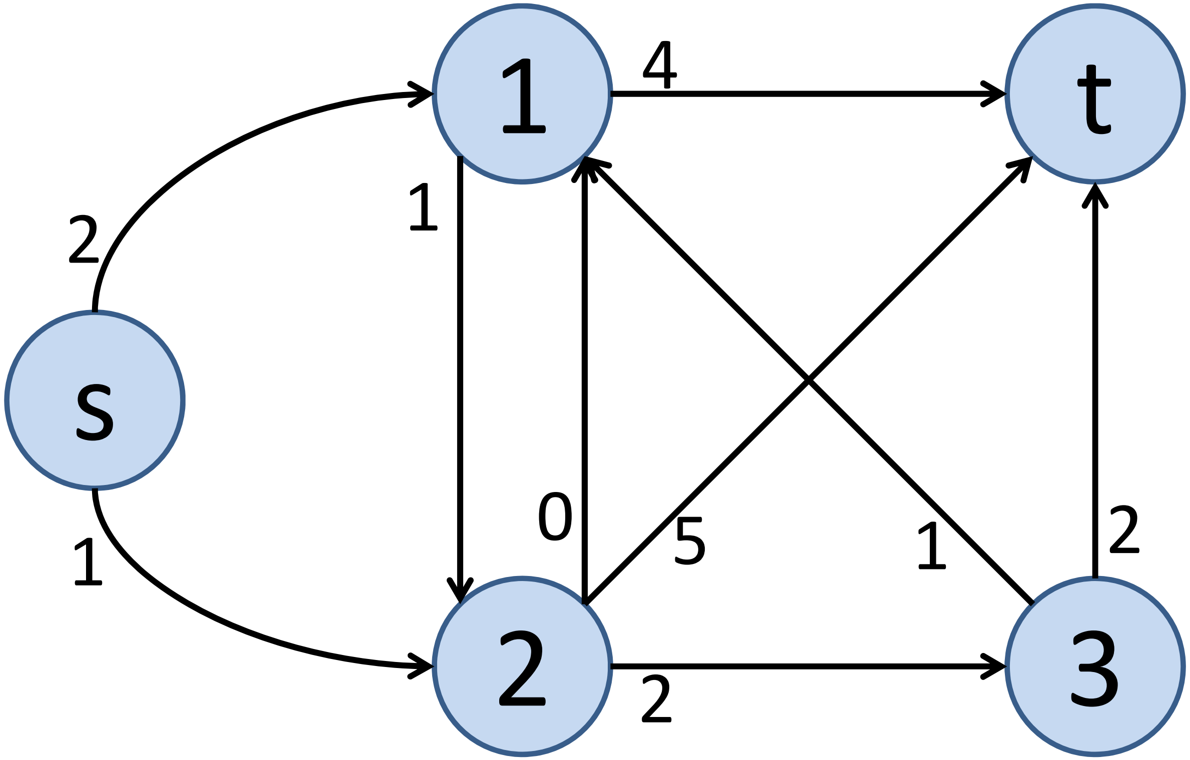
- The integrality theorem for min-cost flow:
  - For any input to the min-cost flow problem where b_i and u_{ij} are integers,
  - then there is an optimal solution with integer values.

Special case #1:
The shortest-path problem

- Input to shortest-path:
  - $G = (N, E)$, a directed graph
  - The node set N contains:
    - A source node, s
    - A sink node, t
  - c_{ij} = length of edge (i, j); $c_{ij} \geq 0$
- Objective: to find a path from s to t with the minimum total distance

Special case #1:
The shortest-path problem

- Example:



Special case #1:
The shortest-path problem

- Step 1:
  Specify the corresponding input to min-cost flow:
  - The same graph $G = (N, E)$ = $G = (N, E)$
  - $b_s = 1$; $b_t = -1$; $b_i = 0$ for all other nodes i in N
  - $c_{ij} = c_{ij}$ (interpret length as cost)
  - $u_{ij} = 1$

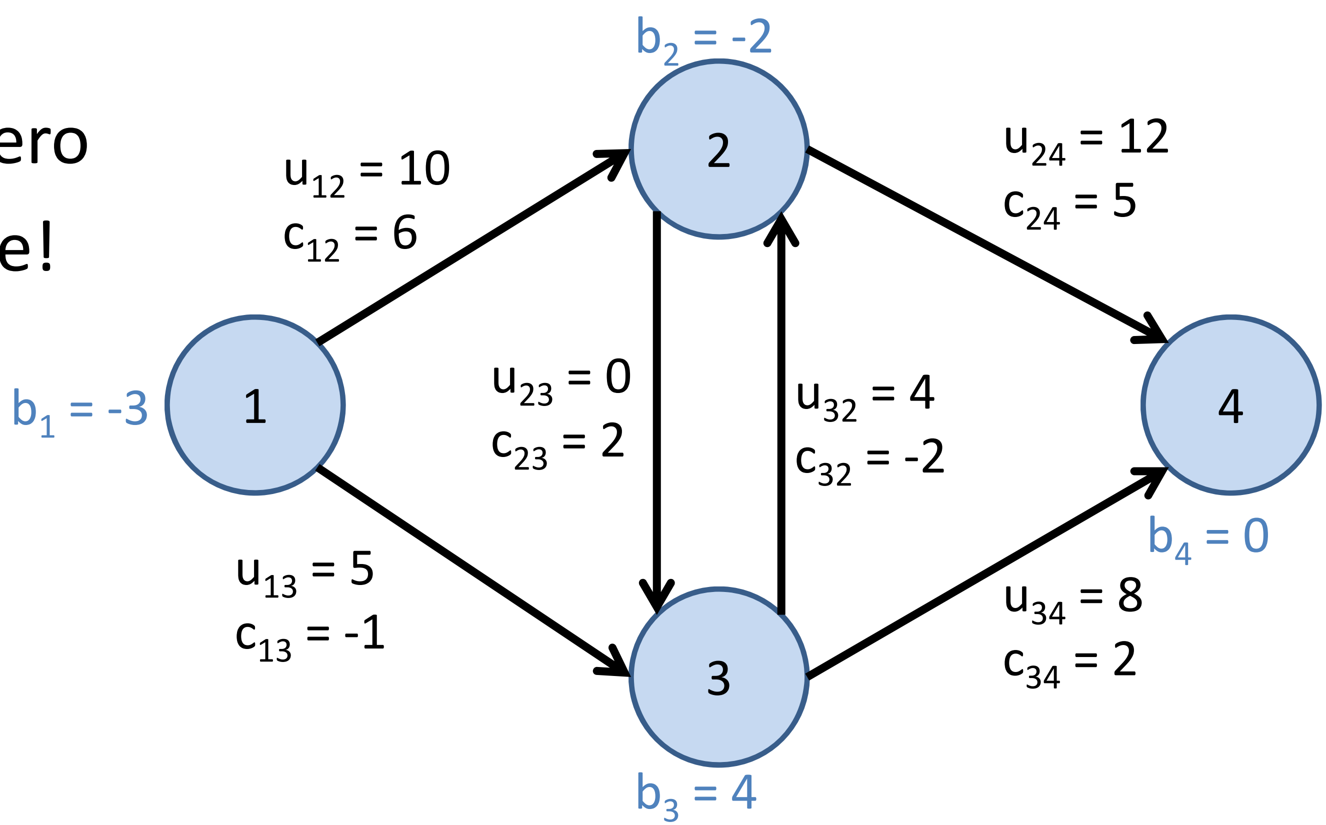
Special case #1:
The shortest-path problem

- Step 2:
  Show correspondence of feasible s-t paths with feasible integer solutions to the min-cost flow problem
- Step 3:
  Show correspondence of shortest s-t paths with optimal solutions to the min-cost flow problem.

Q1 (i>clicker)

Q1: The following is not a valid input to the min-cost flow problem. Why?

A. $c_{13} < 0$
B. There is a negative cycle
C. $u_{23} = 0$
D. $\sum b_i$ is not zero
E. I'm not sure!



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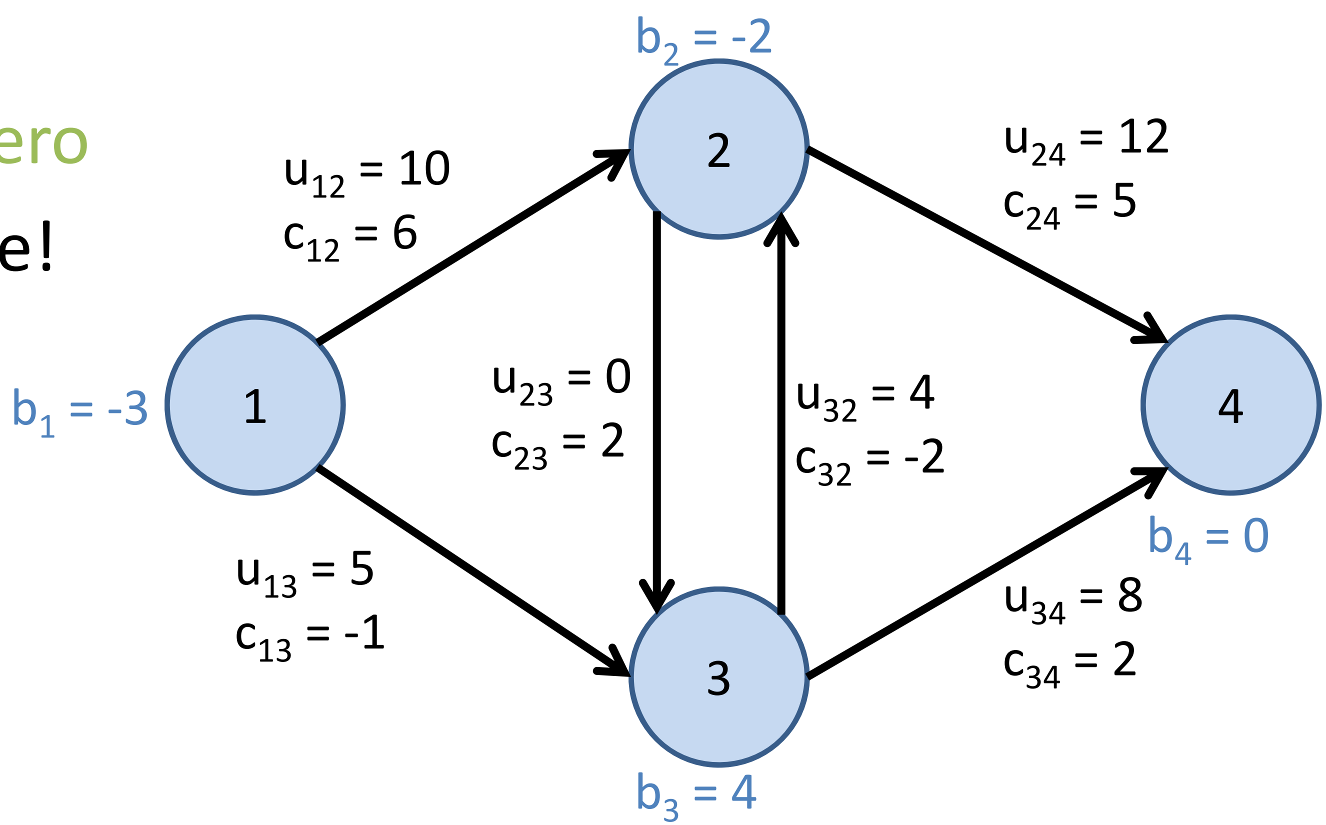


Q2: For node 2, which is the correct flow conservation constraint?

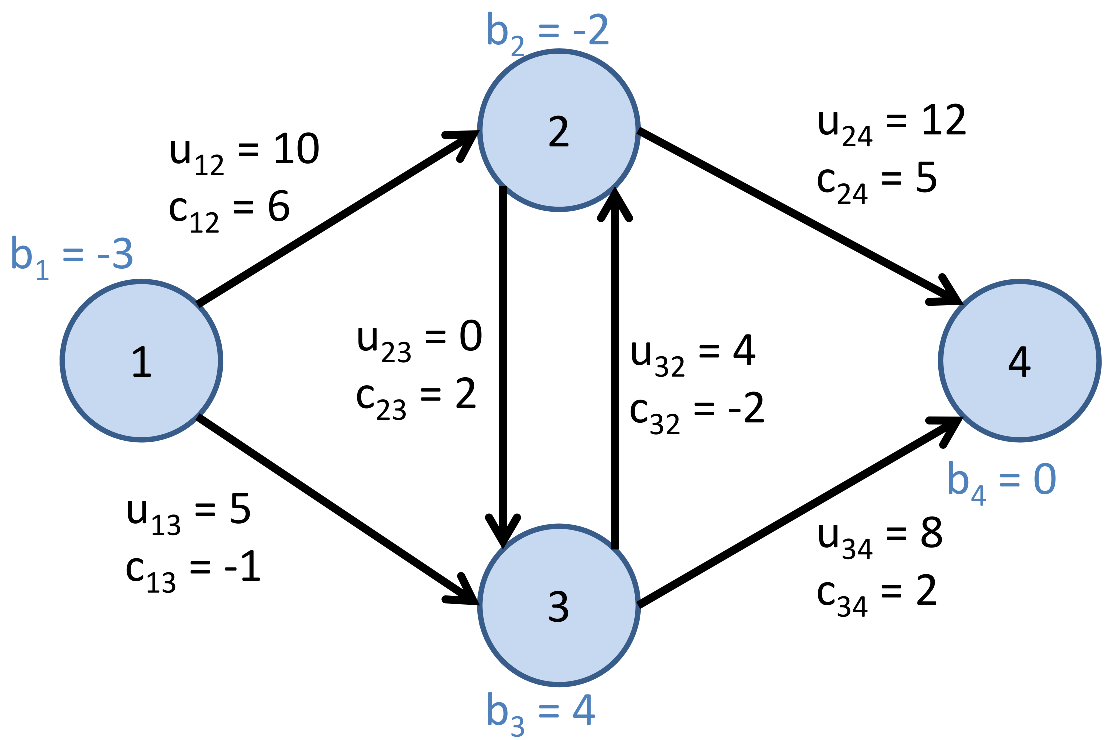
A. $x_{12} + x_{32}$
$- x_{23} - x_{24} = -2$

B. $x_{23} + x_{24}$
$- x_{12} - x_{32} = -2$

C. $x_{12} + x_{32}$
$- x_{23} - x_{24} = 2$

D. $x_{23} + x_{24}$
$- x_{12} - x_{32} = 2$

E. $x_{21} + x_{22}$
$- x_{32} - x_{42} = -2$



Q2: For node 2, which is the correct flow conservation constraint?

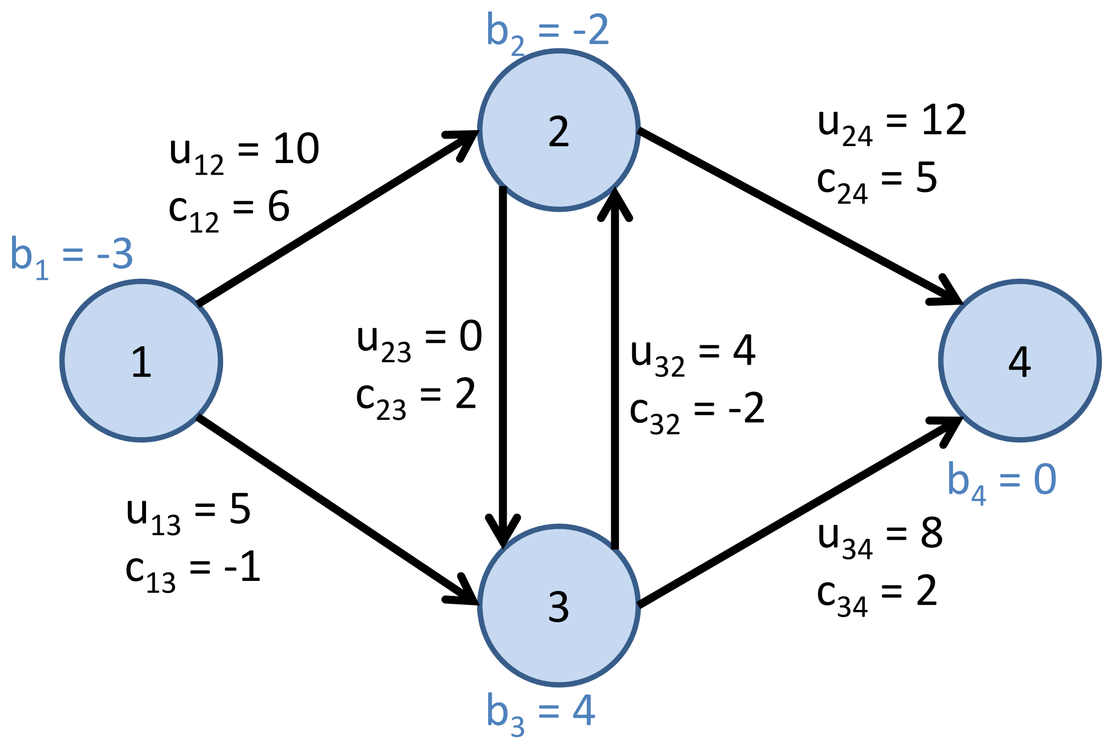
A. $x_{12} + x_{32} - x_{23} - x_{24} = -2$

B. $x_{23} + x_{24} - x_{12} - x_{32} = -2$

C. $x_{12} + x_{32} - x_{23} - x_{24} = 2$

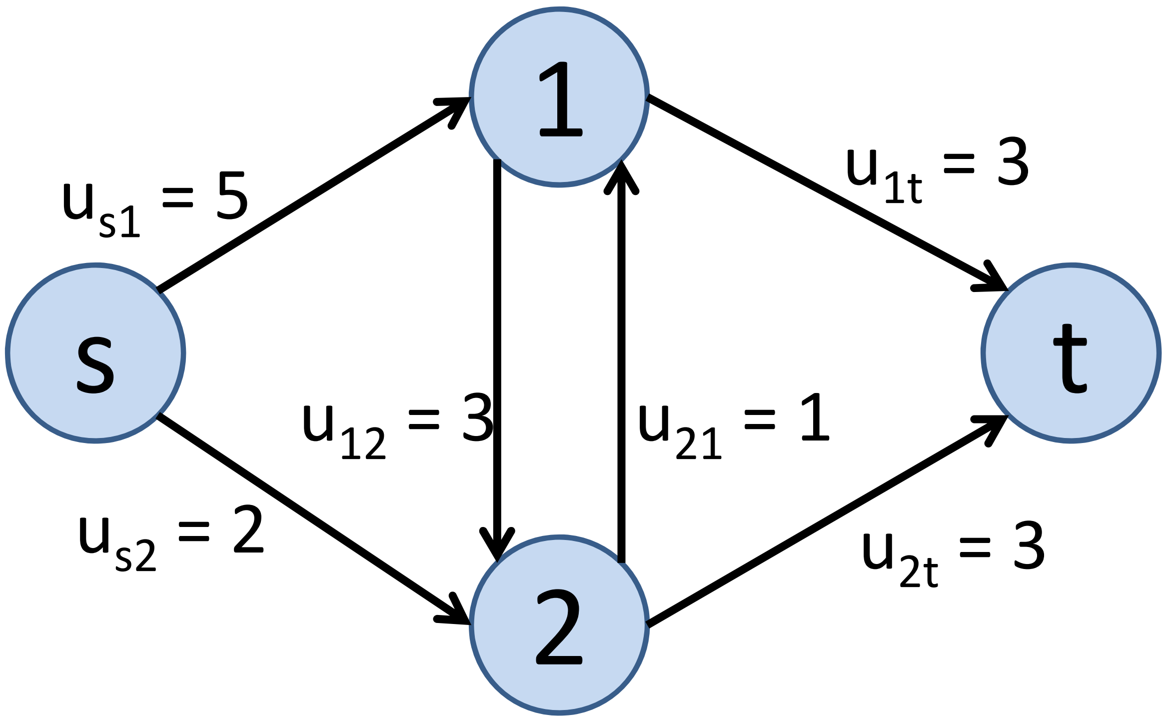
D. $x_{23} + x_{24} - x_{12} - x_{32} = 2$

E. $x_{21} + x_{22} - x_{32} - x_{42} = -2$



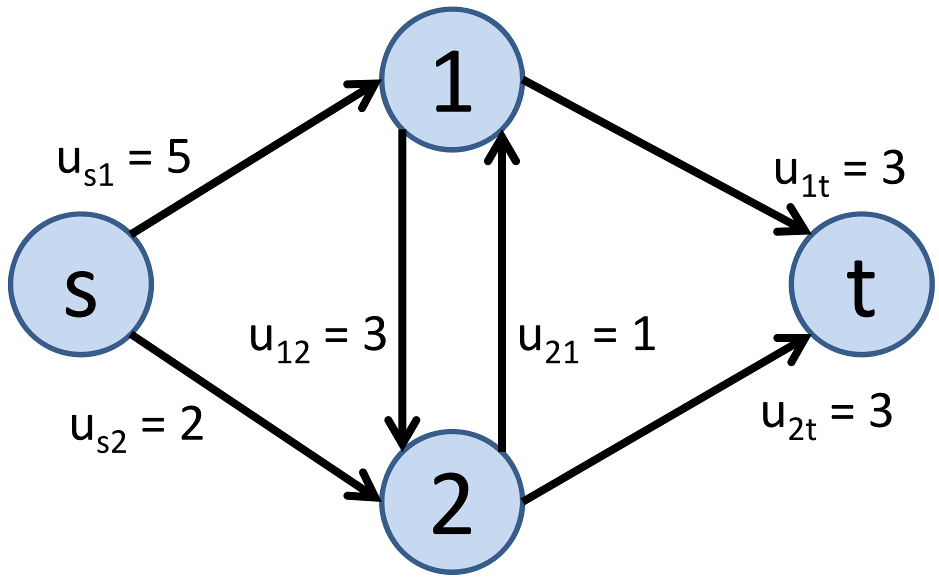
Q3 (i>clicker)

Consider the following maxflow problem



Q3: How many "decision variables" are there in its min-cost flow formulation?

A. 4
B. 5
C. 6
D. 7
E. none of the above



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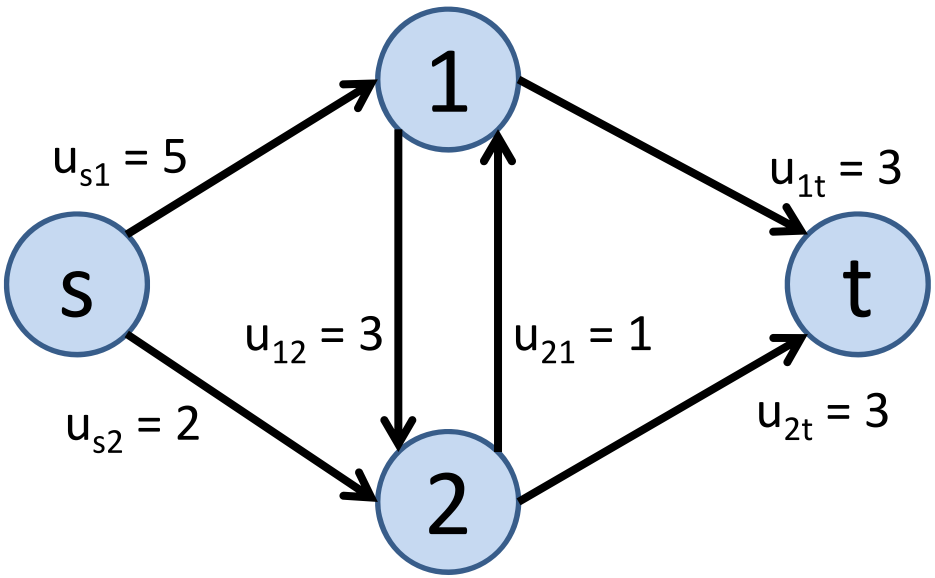
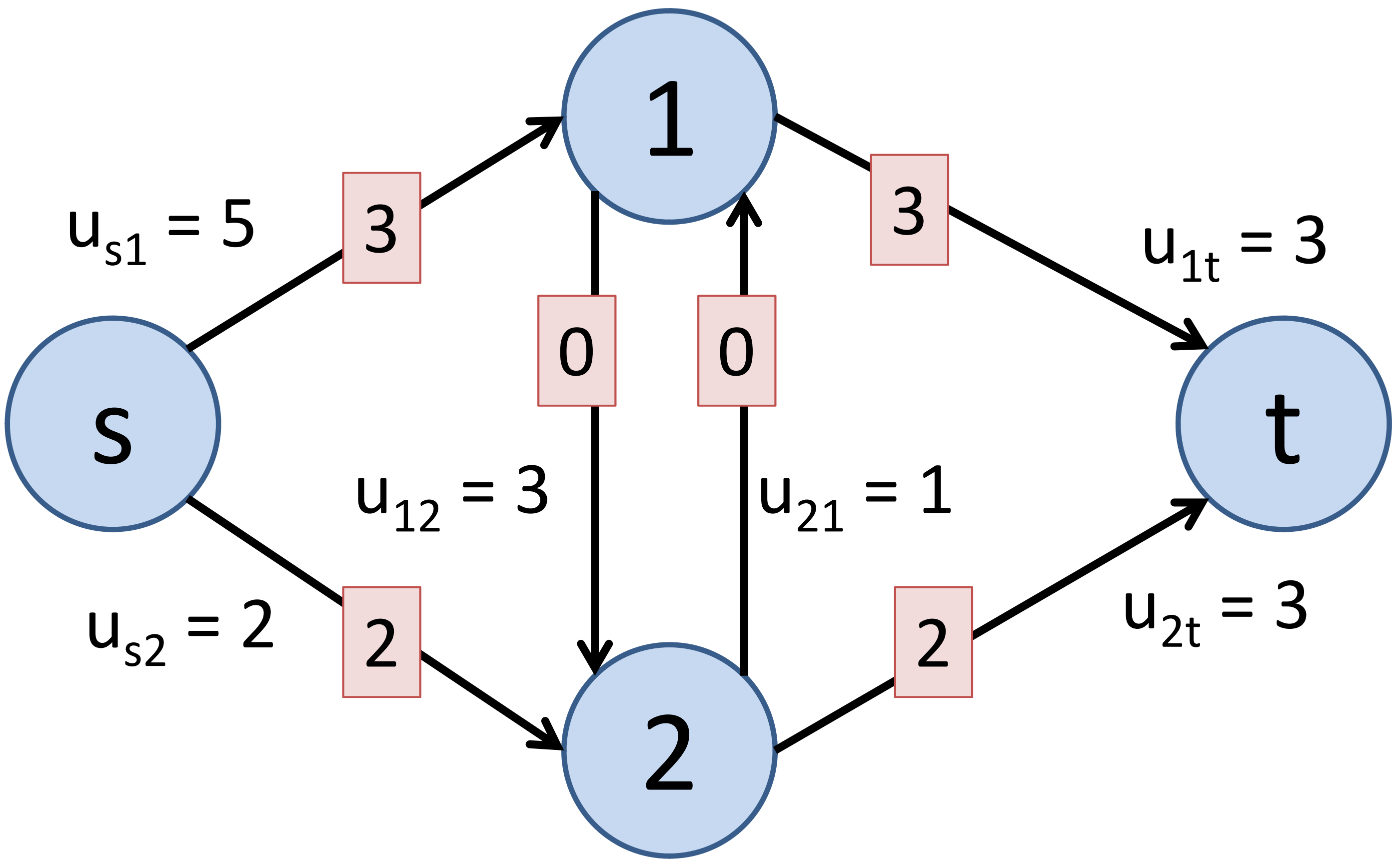
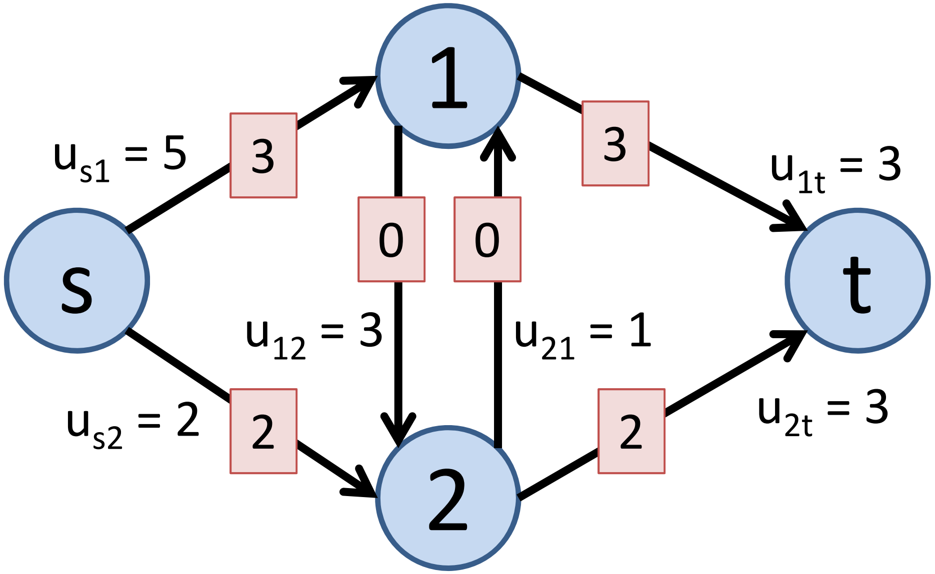


Consider the following feasible solution to maxflow:



Q4: Given a maxflow feasible solution below, what is the corresponding min-cost flow feasible flow on the additional edge?

A. 4
B. 5
C. 6
D. 7
E. none of the above



Q4: Given a maxflow feasible solution below, what is the corresponding min-cost flow feasible flow on the additional edge?

A. 4
B. $5 = x_{ts}$
C. 6
D. 7
E. none of the above