A review of linear programming

Example

CTB sells bagels and cupcakes, earning a profit of \$6 for each dozen of bagels and \$8 for each dozen of cupcakes. We have the following information:

	1 dz Bagels	1 dz Cupcakes	Amount available
Eggs	3	6	50
Flour	7	5	100
Butter	3	4	75

Additionally, CTB must produce at least 3 dozens bagels everyday for its regulars.
How many dozens of bagels and cupcakes should CTB produce each day to maximize total profit?

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$$\text{Total profit} = 6x_1 + 8x_2$$

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$$3x_1 + 6x_2 \leq 50$$

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So, the LP is

$$
\begin{array}{lrrcl}
\max & 6x_1 & +8x_2 & & \\
s.t. & 3x_1 & +6x_2 & \leq & 50 \\
 & 7x_1 & +5x_2 & \leq & 100 \\
 & 3x_1 & +4x_2 & \leq & 75 \\
 & x_1 & & \geq & 3 \\
 & x_1, & x_2, & \geq & 0
\end{array}
$$

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$$
\begin{array}{lrrrcl}
\max & 6x_1 & +8x_2 & & & \\
s.t. & 3x_1 & +6x_2 & + x_3 & = & 50 \\
 & 7x_1 & +5x_2 & & = & 100 \\
 & 3x_1 & +4x_2 & & = & 75 \\
 & x_1 & & & = & 3 \\
 & x_1, & x_2, & x_3, & \geq & 0
\end{array}
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\max & 6x_1 & +8x_2 & & & & \\
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 & x_1 & & & & = & 3 \\
 & x_1, & x_2, & x_3, & x_4, & \geq & 0
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 & 7x_1 & +5x_2 & & + x_4 & & & = & 100 \\
 & 3x_1 & +4x_2 & & & + x_5 & & = & 75 \\
 & x_1 & & & & & - x_6 & = & 3 \\
 & x_1, & x_2, & x_3, & x_4, & x_5, & x_6 & \geq & 0
\end{array}
$$

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So, the LP is (after adding slack variables $x_1, x_2, \ldots, x_6$)

$$
\begin{array}{lrrrrrrcl}
\max & 6x_1 & +8x_2 & & & & & & \\
s.t. & 3x_1 & +6x_2 & + x_3 & & & & = & 50 \\
 & 7x_1 & +5x_2 & & + x_4 & & & = & 100 \\
 & 3x_1 & +4x_2 & & & + x_5 & & = & 75 \\
 & x_1 & & & & & - x_6 & = & 3 \\
 & x_1, & x_2, & x_3, & x_4, & x_5, & x_6 & \geq & 0
\end{array}
$$

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LP in standard form:

$$\begin{aligned} \max \quad & c^T x \\ s.t. \quad & Ax = b \\ & x \geq 0, \end{aligned}$$

where c, x are n-vectors, b is an m-vector, and A is an $m \times n$ matrix.

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In our example, $n = 6$, $m = 4$, $x = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_6 \end{pmatrix}$ and

$$c = \begin{pmatrix} 6 \\ 8 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}, A = \begin{pmatrix} 3 & 6 & 1 & 0 & 0 & 0 \\ 7 & 5 & 0 & 1 & 0 & 0 \\ 3 & 4 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & -1 \end{pmatrix}, b = \begin{pmatrix} 50 \\ 100 \\ 75 \\ 3 \end{pmatrix}.$$