Lecture 25

Lecture 25

Recap from last lecture

2. Constrained Optimization

$$\begin{array}{llll}
\min & f(x) & & \\
s.t. & g_i(x) \geq 0 & \forall i \in \{1,\ldots,p\} & (NLP) \\
 & h_j(x) = 0 & \forall j \in \{1,\ldots,q\} &
\end{array}$$

2. Constrained optimization

<u>Necessary and sufficient condition of optimality</u>

Suppose f, g_i, h_j are continuous , differentiable.

If $\bar{x}$ is an optimal solution for (NLP), then there exists $y_i \geq 0, z_j$ that satisfy :

- If $g_i(\bar{x}) > 0$, then $y_i = 0$,
- and $\nabla L_{y,z}(\bar{x}) = 0$

2. Constrained optimization

Necessary and sufficient condition of optimality

Suppose f, g_i, h_j are continuous , differentiable.

If $\bar{x}$ is an optimal solution for (NLP), then

there exists $y_i \geq 0, z_j$ that satisfy :

- If $g_i(\bar{x}) > 0$, then $y_i = 0$,

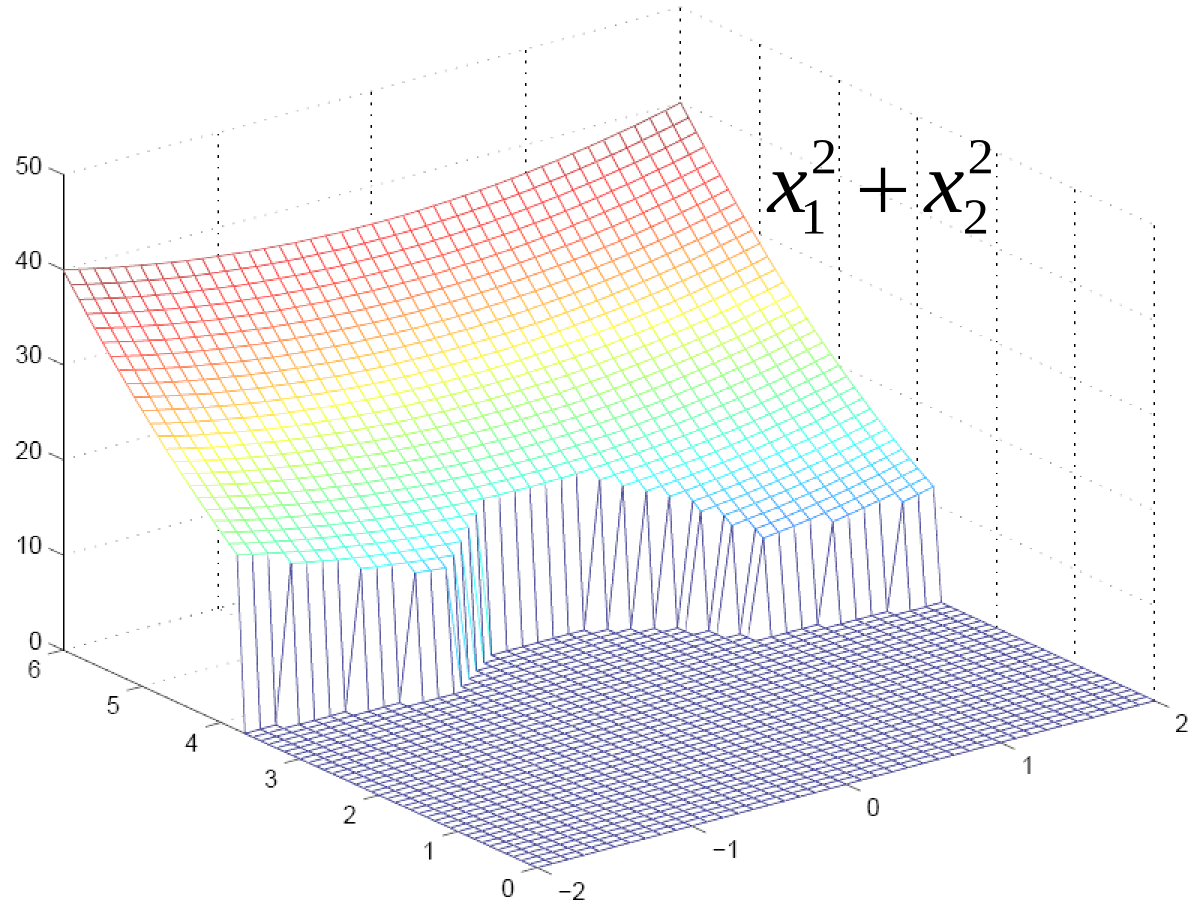
- and $\nabla f - \sum_{i=1}^{p} y_i g_i(\bar{x}) - \sum_{j=1}^{q} z_j h_j(\bar{x}) = 0$

Example:

Nonlinear constrained optimization

$$
\begin{array}{lll}
\min & x_1^2 + x_2^2 & \\
s.t. & x_1^2 + x_2 & \geq 4 \\
 & x_1 + 4x_2 & \geq 13
\end{array}
$$

Example:
Nonlinear constrained optimization



Example:
Nonlinear constrained optimization

$$
\begin{aligned}
\min \quad & x_1^2 + x_2^2 & \\
s.t. \quad & x_1^2 + x_2 & \geq 4 \\
& x_1 + 4x_2 & \geq 13
\end{aligned}
$$

Example:
Nonlinear constrained optimization

$$\begin{aligned} \min \quad & f(x) \\ s.t. \quad & g_1(x) \geq 0 \\ & g_2(x) \geq 0 \end{aligned}$$

$$\begin{aligned} f(x) &= x_1^2 + x_2^2 \\ g_1(x) &= x_1^2 + x_2 - 4 \\ g_2(x) &= x_1 + 4x_2 - 13 \end{aligned}$$

Example:
Nonlinear constrained optimization

$$L_y(x) = f(x) - y_1 g_1(x) - y_2 g_2(x)$$

$$\nabla L_y(x) = \nabla f(x) - y_1 \nabla g_1(x) - y_2 \nabla g_2(x)$$

$$\begin{aligned} f(x) &= x_1^2 + x_2^2 \\ g_1(x) &= x_1^2 + x_2 - 4 \\ g_2(x) &= x_1 + 4x_2 - 13 \end{aligned}$$

Q: Does the point x=(10, 20.25) satisfy the necessary optimality conditions?

$$\begin{aligned} \min \quad & f(x) & \\ s.t. \quad & g_1(x) & \geq 0 \\ & g_2(x) & \geq 0 \end{aligned}$$

$$\begin{aligned} f(x) &= x_1^2 + x_2^2 \\ g_1(x) &= x_1^2 + x_2 - 4 \\ g_2(x) &= x_1 + 4x_2 - 13 \end{aligned}$$

2. Constrained optimization

<u>Necessary and sufficient condition of optimality</u>

Suppose f, g_i, h_j are continuous , differentiable.

If $\bar{x}$ is an optimal solution for (NLP), then there exists $y_i \geq 0, z_j$ that satisfy :

- If $g_i(\bar{x}) > 0$, then $y_i = 0$,
- and $\nabla L_{y,z}(\bar{x}) = 0$

i>clicker

Q: Does the point $x=(10, 20.25)$ satisfy the necessary optimality conditions?

A. Yes

B. No

C. I'm not sure

Lecture 25

Newton’s Method

1. Unconstrained optimization

$$\text{Min } f(x)$$

where

- $x = (x_1, x_2, \ldots, x_n)$
- $f(x)$ is continuous and differentiable

1. Unconstrained optimization

<u>Necessary condition of optimality</u>

If $\bar{x}$ is a global minimizer of f, then

$$\nabla f(\bar{x}) = 0.$$

1. Unconstrained optimization

Example:

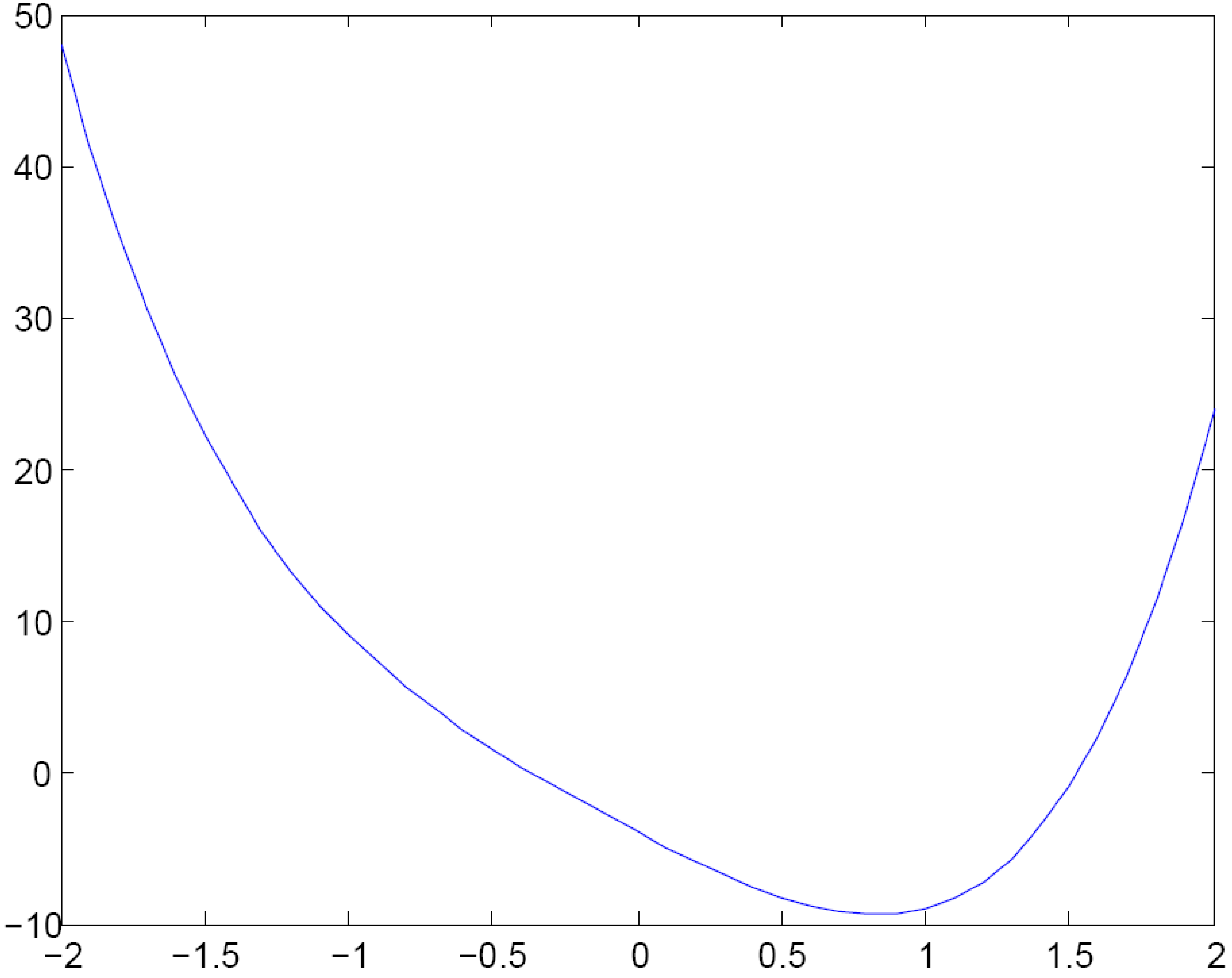
$$f(x) = 2x^2 - 10x - 5.5$$

1. Unconstrained optimization

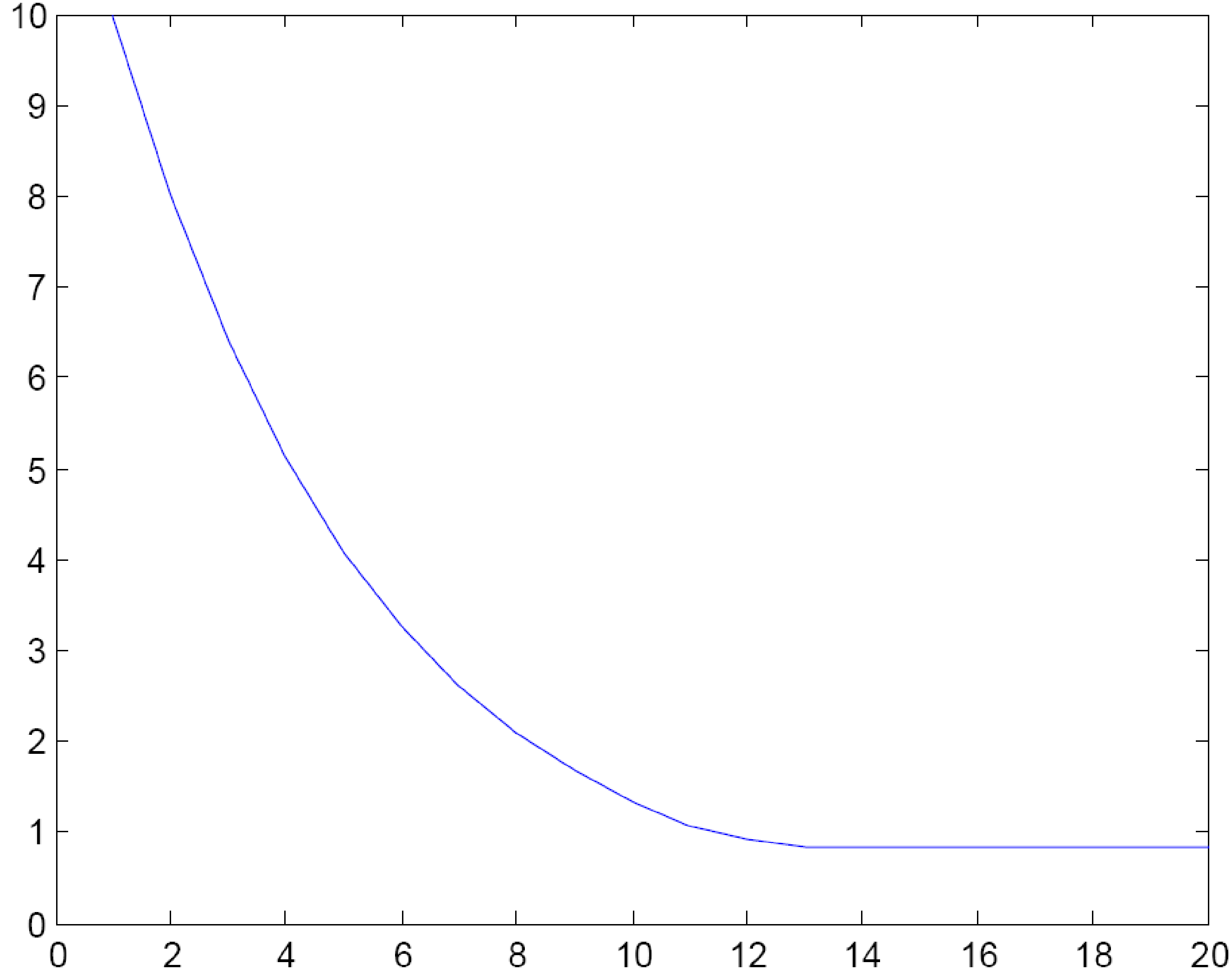
Example:

$$f(x) = 2x^6 + x^3 + 2x^2 - 10x - 5.5$$

Newton's Method



Newton's Method



1. Unconstrained optimization

Example:

$$f(x) = 2x_1^4 + 2.5x_1^2 - 2x_1x_3 + 2x_2^2 + 1.5x_3^2$$
$$-3x_1 + x_2 - 2x_3 - 2$$

1. Unconstrained optimization