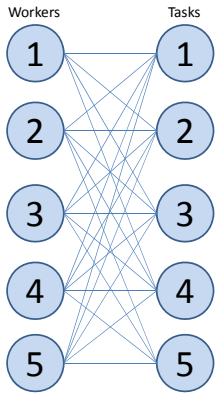
Lecture 8: The Assignment Problem Handout

3	5	6	2	2
2	3	5	3	2
3	0	4	2	2
3	0	3	2	2
0	3	0	1	0

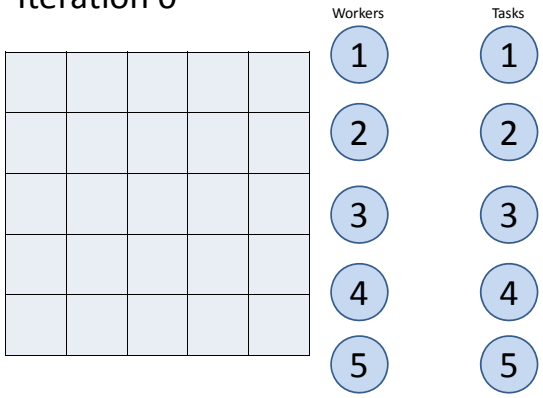


The Hungarian Algorithm

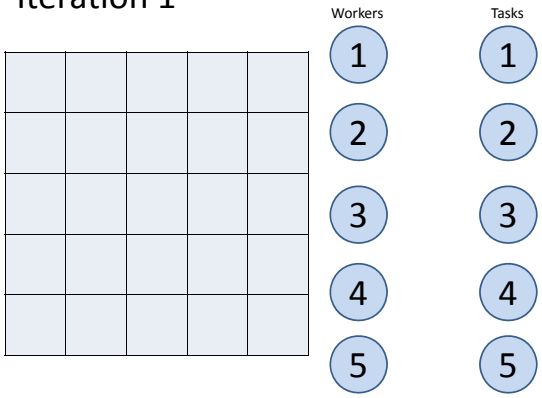
0. Set up a cost table. Assume all costs are ≥ 0.
1. For each row i = 1, …, n
   let α_i = the smallest entry in row i
   update: $t_{ij} \leftarrow t_{ij} - \alpha_i$ for all entries (i,j) in row i
2. For each col j = 1, …, n
   let β_j = the smallest entry in column j
   update: $t_{ij} \leftarrow t_{ij} - \beta_j$ for all entries (i,j) in col j
3. Check if there is an all-zero assignment
   If there is one, this assignment is optimal
   If there isn't one:
   - find a zero-cover containing fewer than n lines,
   - let δ = the smallest entry not covered by any lines
   - update:
     $t_{ij} \leftarrow t_{ij} - \delta$ if entry (i, j) is not covered by any lines
     $t_{ij} \leftarrow t_{ij}$ if entry (i, j) is covered by exactly one line
     $t_{ij} \leftarrow t_{ij} + \delta$ if entry (i, j) is covered by exactly two lines

   (Repeat step 3 until there is an all-zero assignment)

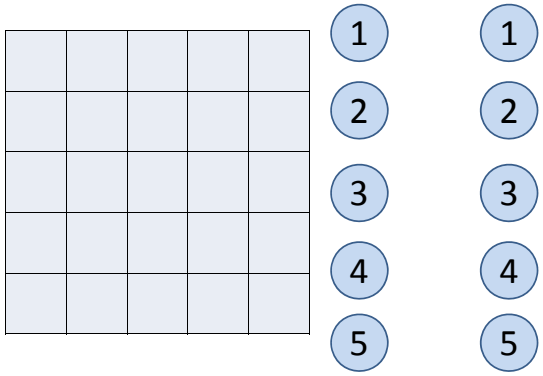
Iteration 0



Iteration 1



Iteration 2



Iteration 3