Lecture 4

Previously in Opt 2 …

Special case #3:
The maximum-flow problem

- Input
  - $G = (N, E)$, a directed graph
  - The node set N contains:
    - A source node, s
    - A sink node, t
  - u_{ij} = edge capacity; $u_{ij} \geq 0$
- Objective: To maximize net flow into t, subject to constraints:
  - Capacity constraints
  - Flow conservation constraints:

    Net flow out of node i = 0

(for each node i in N, <u>except s and t</u>)

Special case #3:
The maximum-flow problem

- Input
  - $G = (N, E)$, a directed graph
  - The node set N contains:
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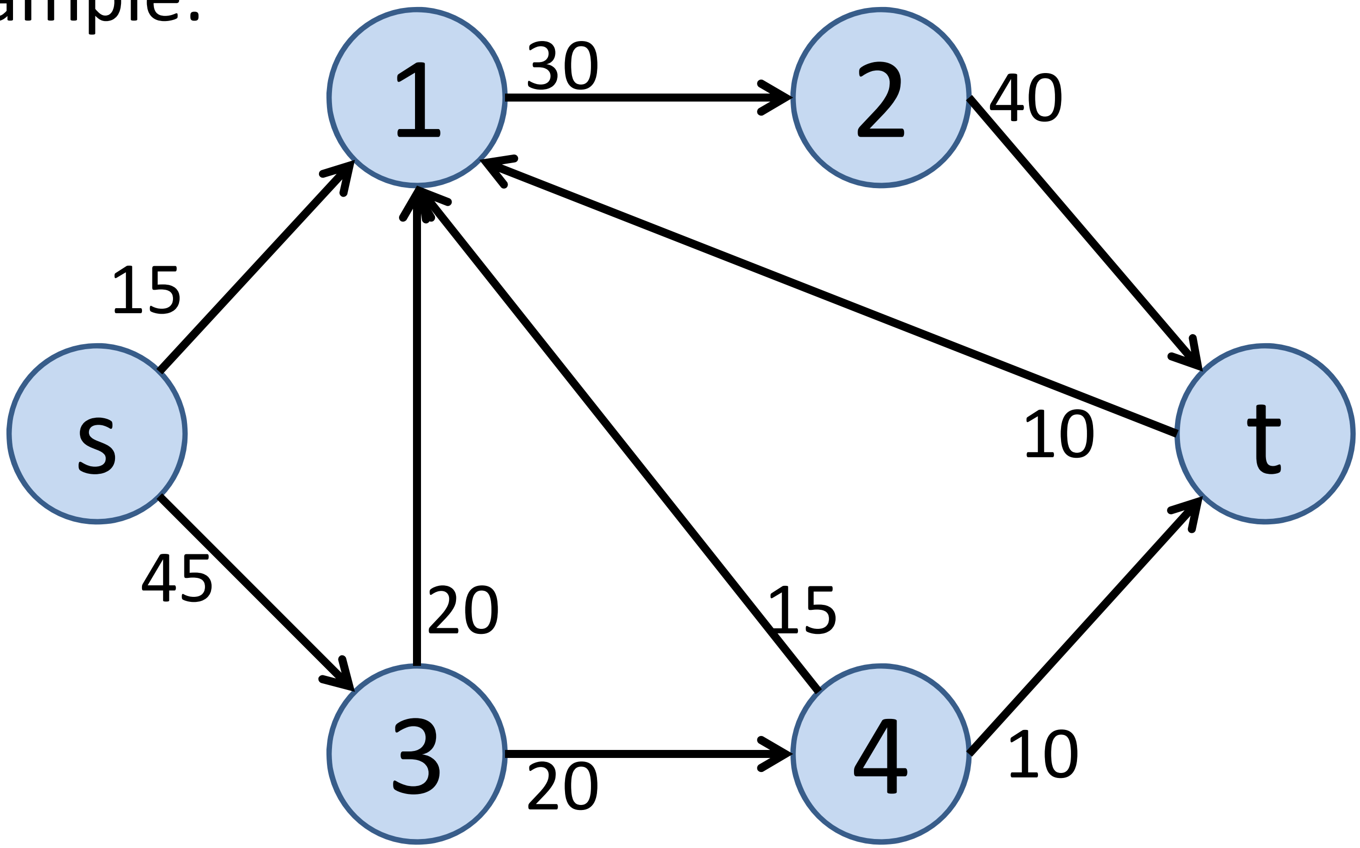
  - u_{ij} = edge capacity; $u_{ij} \geq 0$
- Objective: To maximize net flow into t, subject to constraints:
  - Capacity constraints
  - Flow conservation constraints:

    Total flow into i = total flow out of i

(for each node i in N, <u>except s and t</u>)

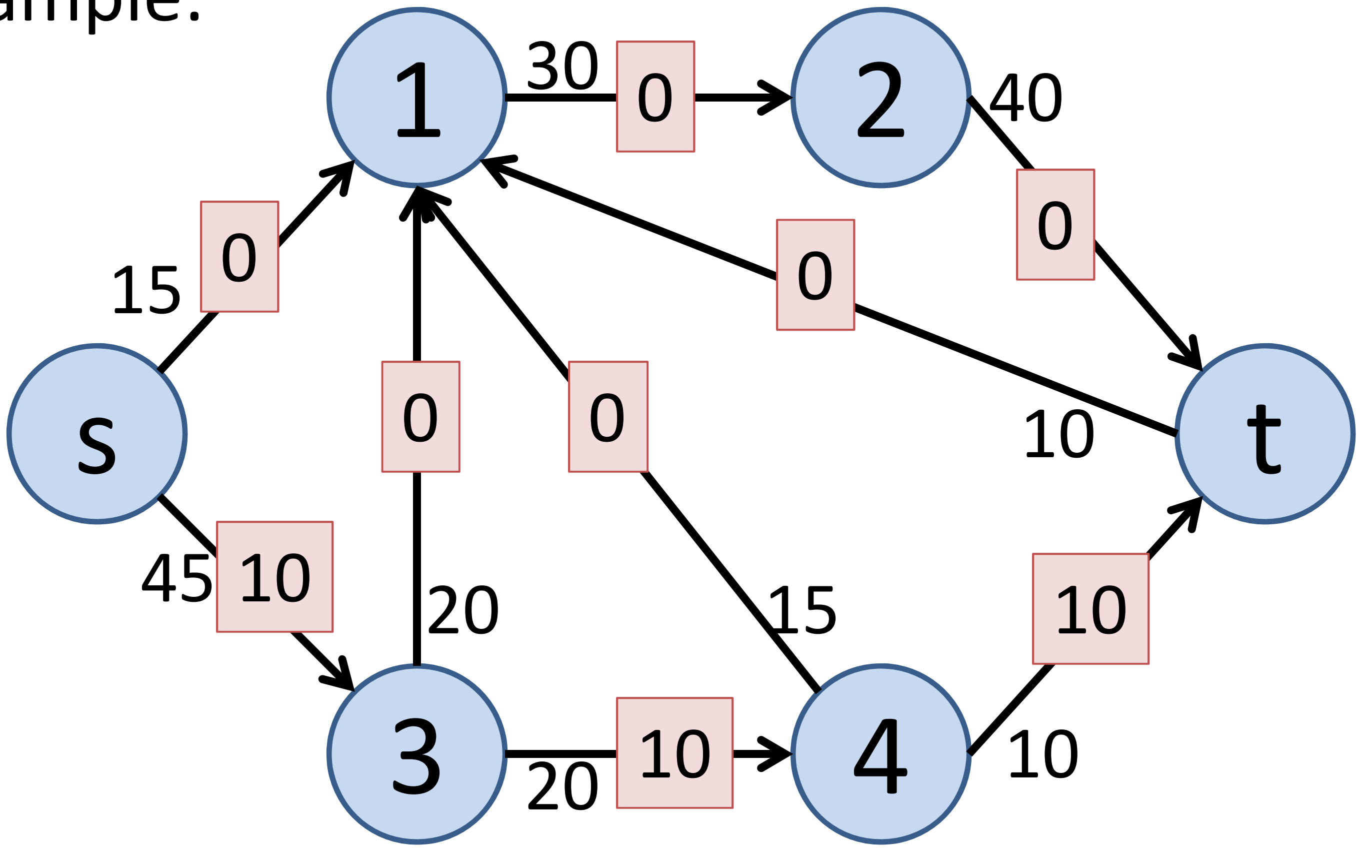
Special case #3: The maximum-flow problem

- Example:



Special case #3:
The maximum-flow problem

- Example:

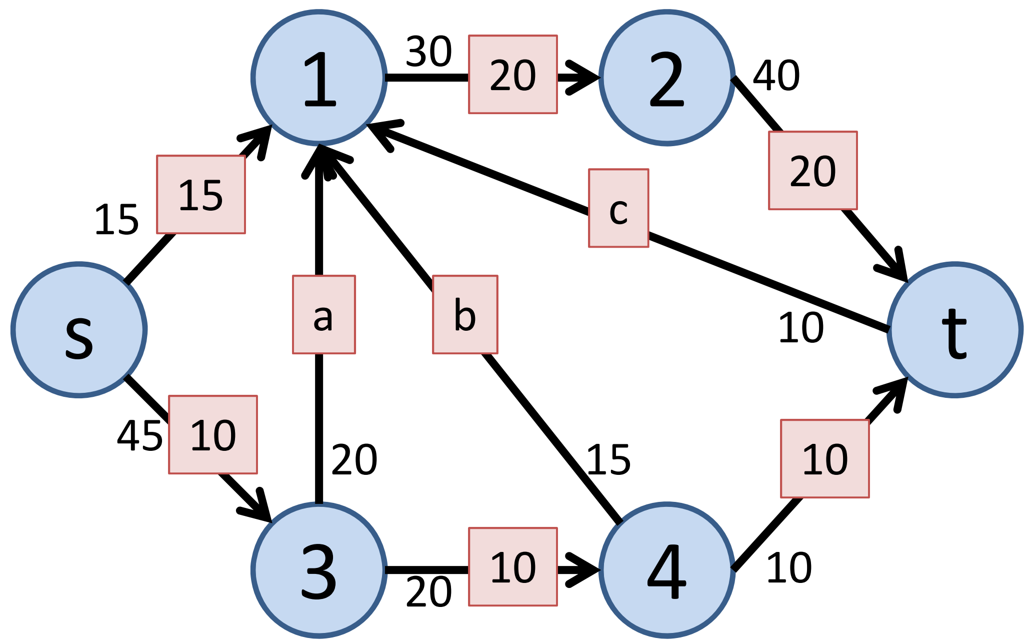


Flow value = 10

Q1 (i>clicker)

Q1: What are a, b, c so that we have a feasible solution? What is the corresponding flow value?

A. a = 0, b= 0 , c = 0, flow value = 30
B. a = 0, b = 0, c = 0, flow value = 25
C. a = 0, b = 0, c = 5, flow value = 30
D. a = 0, b = 0, c = 5, flow value = 25
E. a = 0, b = 0, c = 5,
flow value
= 40*20 + 10 * 10
= 900



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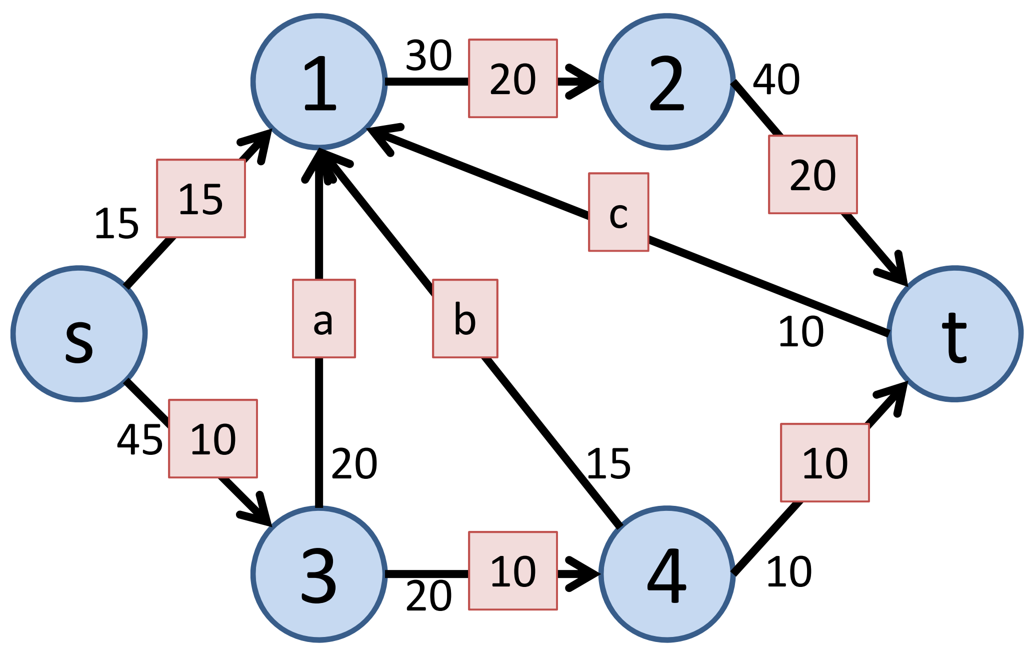
E. a = 0, b = 0, c = 5, flow value = 40*20 + 10 * 10 = 900



The maximum-flow problem compared to mincost flow

Maxflow

- Input
  - $G = (N, E)$, a directed graph
  - The node set N contains:
    - A source node, s
    - A sink node, t
  - u_{ij} = edge capacity; $u_{ij} \geq 0$
- Objective:
  To maximize net flow into t, subject to constraints:
  - Capacity constraints
  - Flow conservation constraints:

    Net flow out of node i = 0

    (for each node i in N, except s and t)

Mincost flow

- Input
  - $G = (N, E)$, a directed graph
  - b_i = node supply values, where $\sum b_i = 0$.
  - c_{ij} = edge costs
  - u_{ij} = edge capacity, $u_{ij} \geq 0$
- Objective: To minimize total cost, subject to constraints:
  - Capacity constraints
  - Flow conservation constraints:

    Net flow out of node i = b_i

    (For each node i in N)

Step 1: Specify the corresponding input to mincost flow

Maxflow

- Input
  - $G = (N, E)$, a directed graph
  - The node set N contains:
    - A source node, s
    - A sink node, t
  - u_{ij} = edge capacity; $u_{ij} \geq 0$

Mincost flow

- Input:

Step 1: Specify the corresponding input to mincost flow

Maxflow

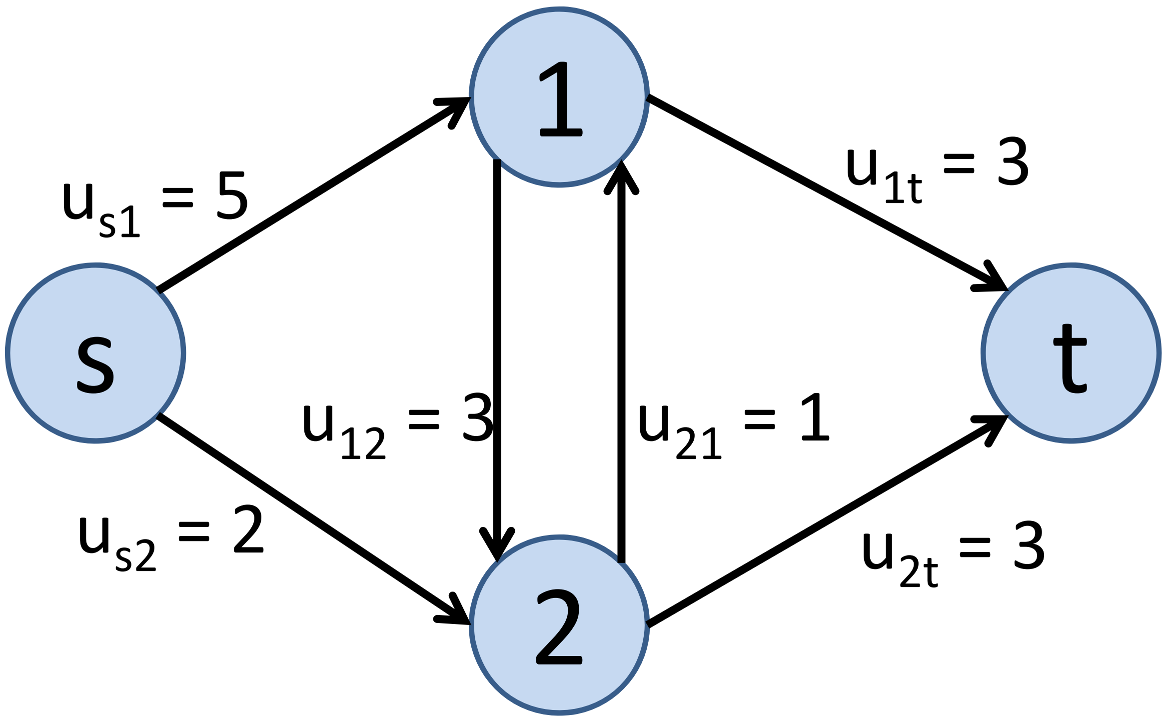
- Input
  - $G = (N, E)$, a directed graph
  - The node set N contains:
    - A source node, s
    - A sink node, t
  - u_{ij} = edge capacity; $u_{ij} \geq 0$

Mincost flow

- Input
  - $G' = (N', E')$ where
    - $N' = N$
    - $E' = E \cup \{(t, s)\}$
  - $u'_{ij} = u_{ij}$ for (i, j) in E; $u'_{ts} = +\infty$
  - $c'_{ij} = 0$ for (I, j) in E; $c'_{ts} = -1$
  - $b'_{i} = 0$ for all I in N'

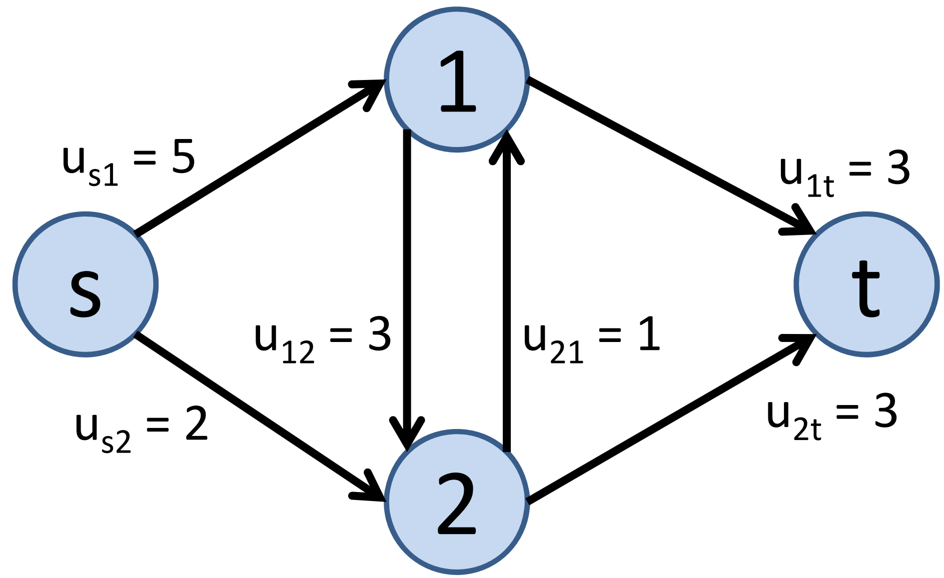
Q2 (i>clicker)

Consider the following maxflow problem



Q2: How many “decision variables” are there in its min-cost flow formulation?

A. 4
B. 5
C. 6
D. 7
E. none of the above



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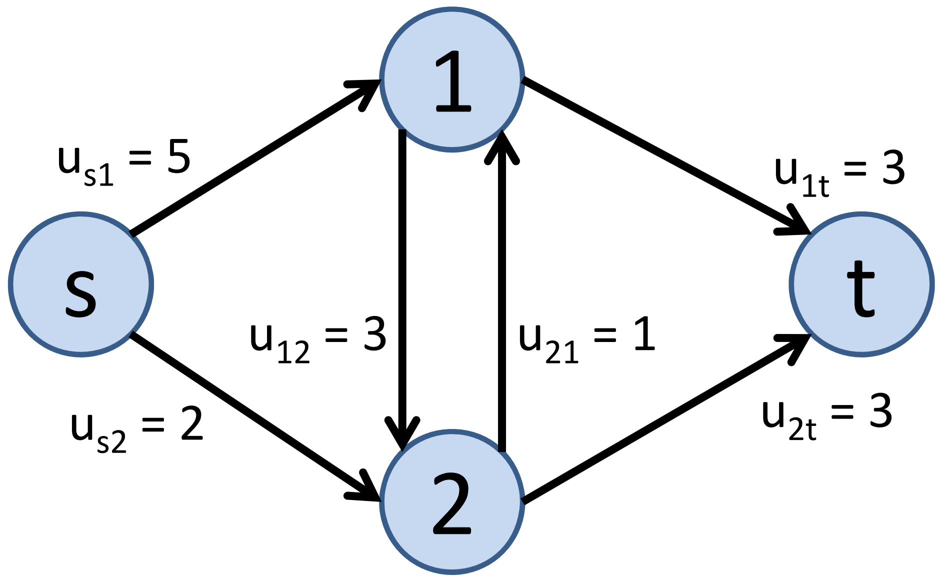
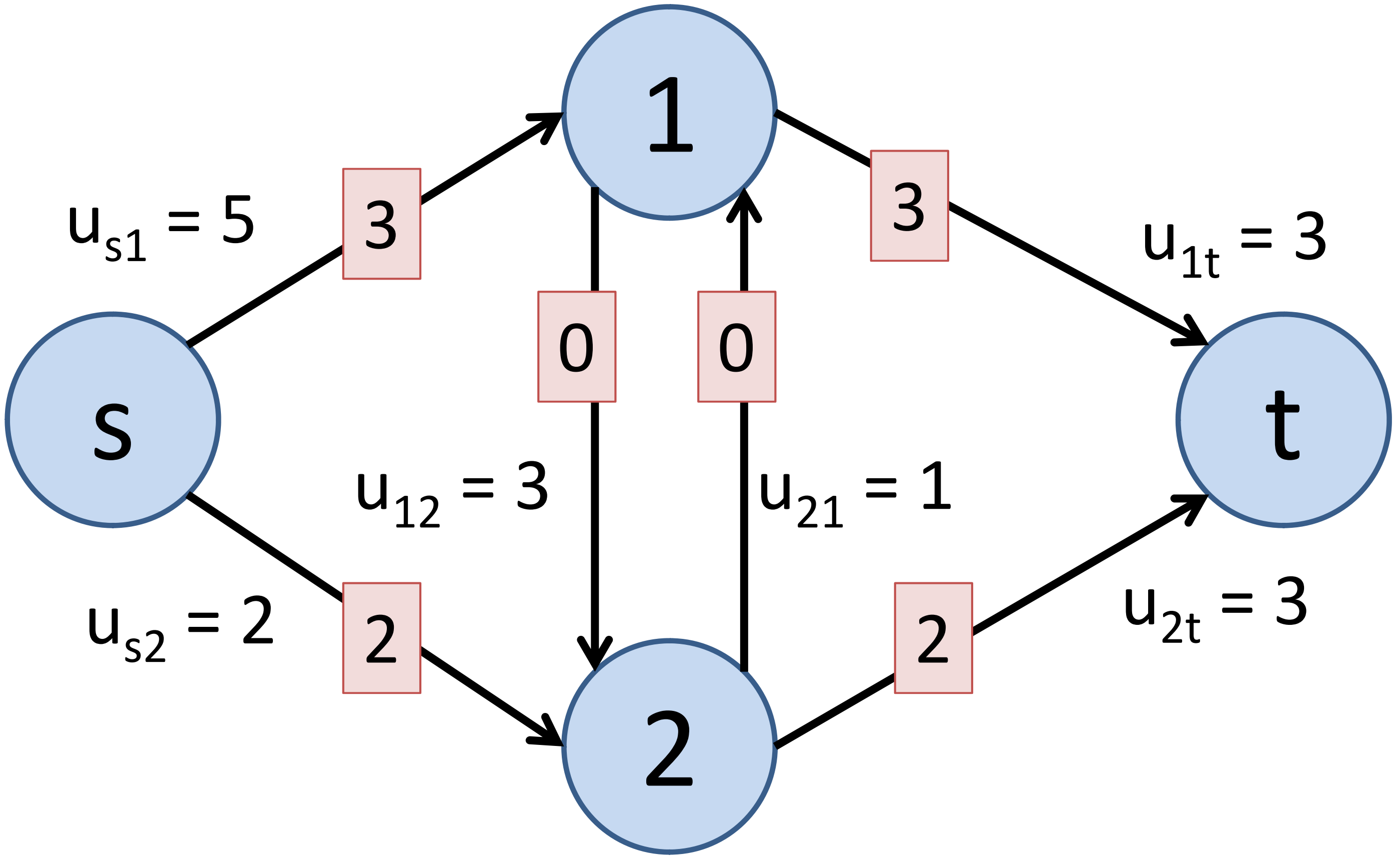


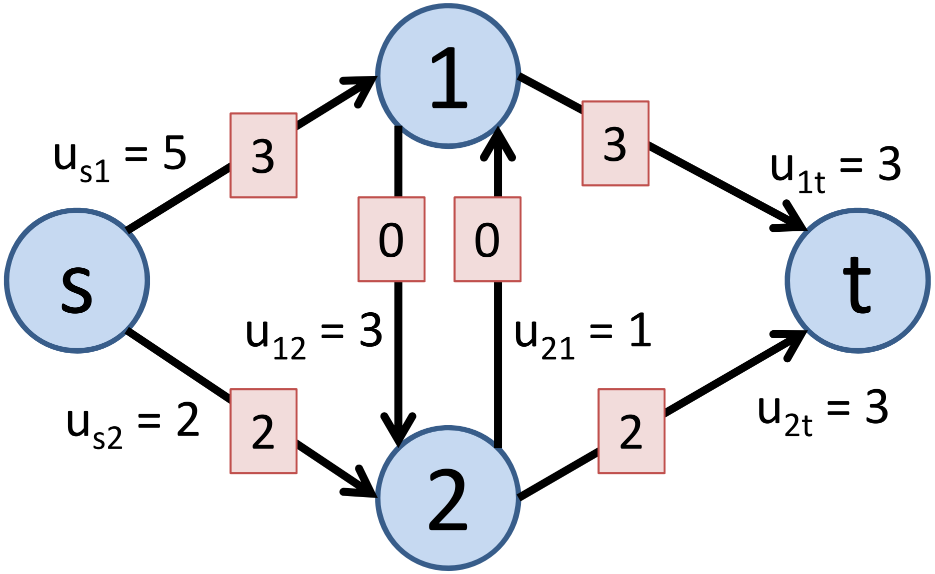
Q3 (i>clicker)

Consider the following feasible solution to maxflow:



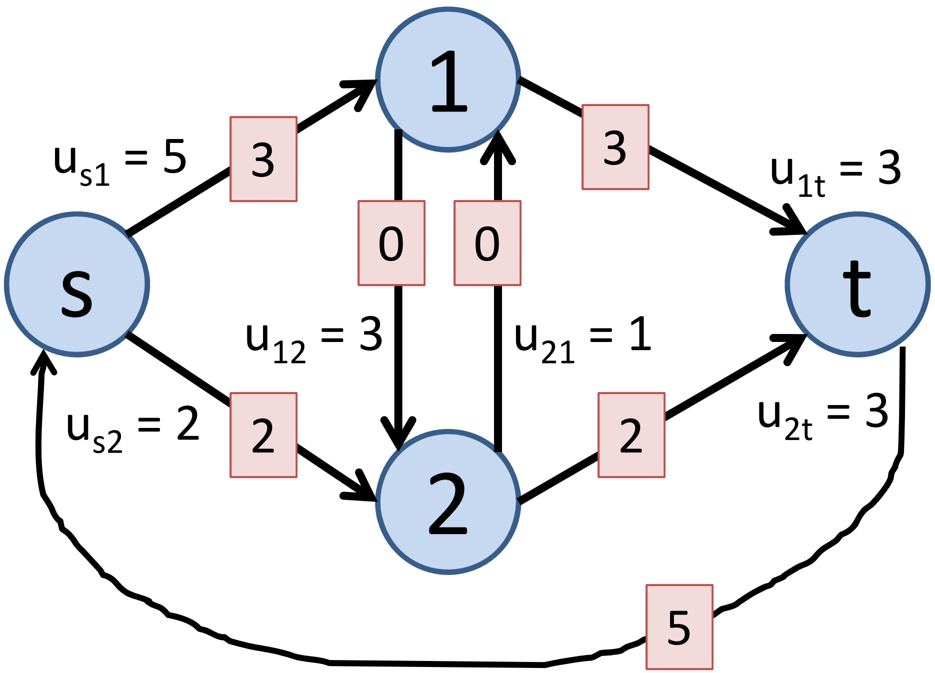
Q3: Given a maxflow feasible solution below, what is the corresponding min-cost flow feasible flow on the additional edge?

A. 4
B. 5
C. 6
D. 7
E. none of the above



Q3: Given a maxflow feasible solution below, what is the corresponding min-cost flow feasible flow on the additional edge?

A. 4
B. $5 = x_{ts}$
C. 6
D. 7
E. none of the above



Step 2: Show that there is a correspondence of feasible solutions

Maxflow

Mincost flow