Lecture 24

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Recap from last lecture

1. Unconstrained optimization

$$\text{Min } f(x)$$

where

- $x = (x_1, x_2, \ldots, x_n)$
- $f(x)$ is continuous and differentiable

1. Unconstrained optimization

<u>Necessary condition of optimality</u>

If $\bar{x}$ is a global minimizer of *f*, then

$$\nabla f(\bar{x}) = 0.$$

1. Unconstrained optimization

<u>Necessary and sufficient condition of optimality</u>

Suppose f is continuous, differentiable, and convex.

$\bar{x}$ is a global minimizer of f <u>if and only if</u>

$$\nabla f(\bar{x}) = 0.$$

Convex functions

Definition (convex function)

A function f is convex if:

for any two points a and b, and any number λ with $0 \leq \lambda \leq 1$,

the following inequality is satisfied:

$$f(\lambda a + (1- \lambda)b) \leq \lambda f(a) + (1- \lambda)f(b).$$

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Q: Which functions are convex?

A. f_1 only

B. f_1 and f_2

C. f_1, f_2, and f_3

D. f_1, f_2, f_3, and f_4

E. f_1, f_2, f_3, f_4, and f_5

Q: Which functions are convex?

A. f_1 only

B. f_1 and f_2

C. f_1, f_2, and f_3

D. f_1, f_2, f_3, and f_4

E. f_1, f_2, f_3, f_4, and f_5

Lecture 24

2. Constrained Optimization

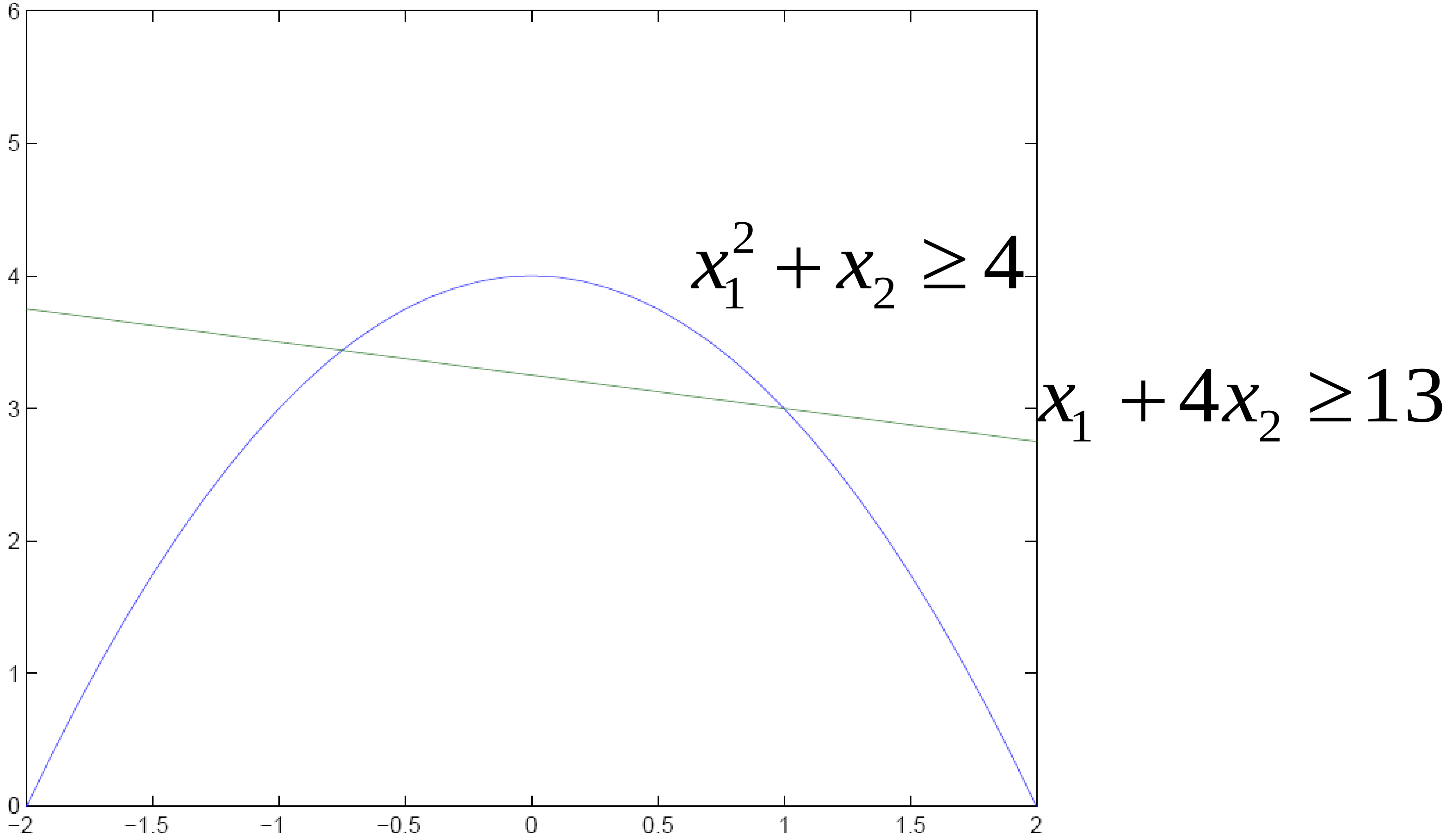
$$
\boxed{\begin{array}{llll}
\min & f(x) & & \\
s.t. & g_i(x) & \geq 0 & \forall i \in \{1,\ldots,p\} \quad (NLP) \\
 & h_j(x) & = 0 & \forall j \in \{1,\ldots,q\}
\end{array}}
$$

Example:

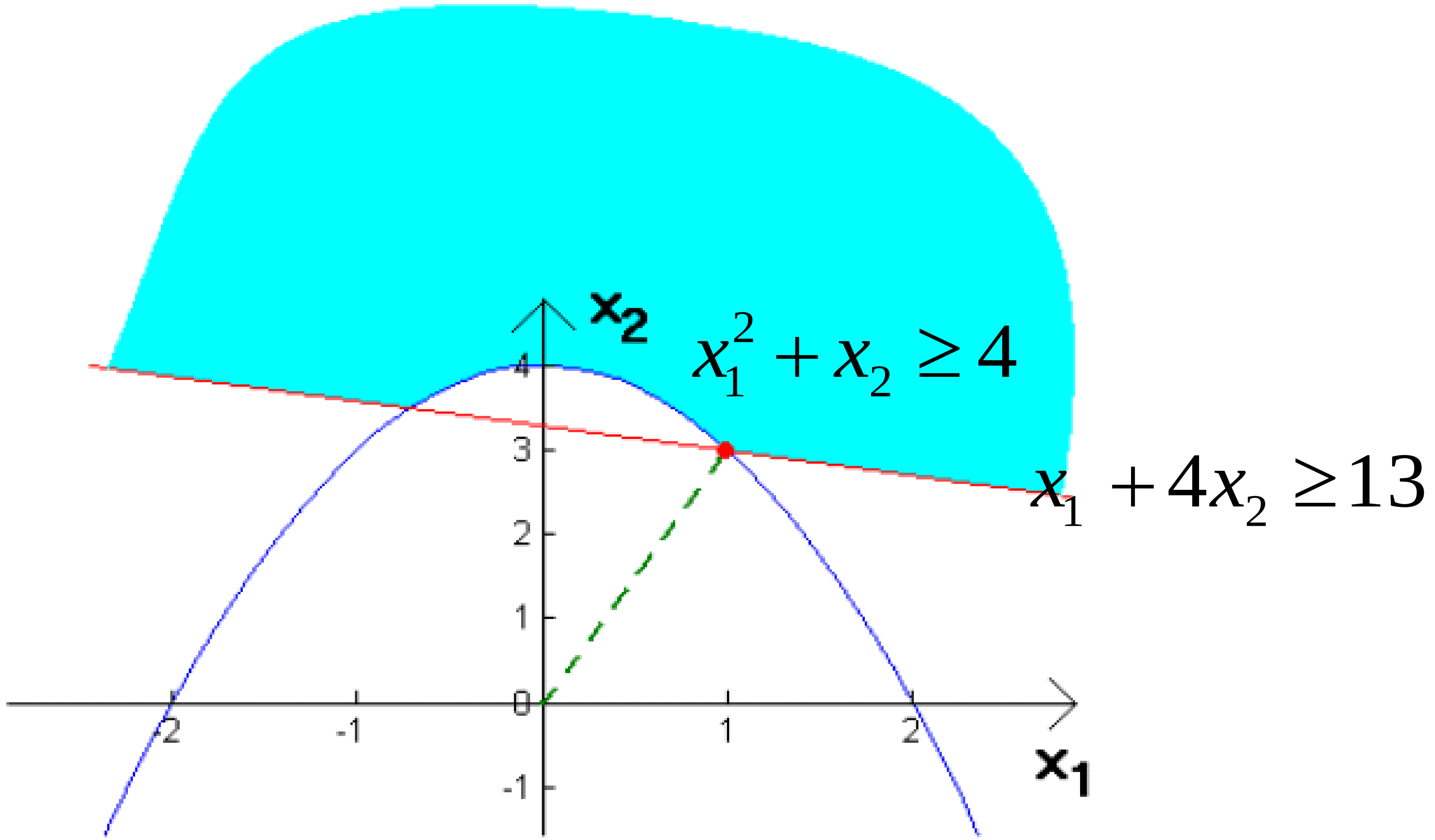
Nonlinear constrained optimization

$$\begin{aligned} \min \quad & x_1^2 + x_2^2 & \\ s.t. \quad & x_1^2 + x_2 & \geq 4 \\ & x_1 + 4x_2 & \geq 13 \end{aligned}$$

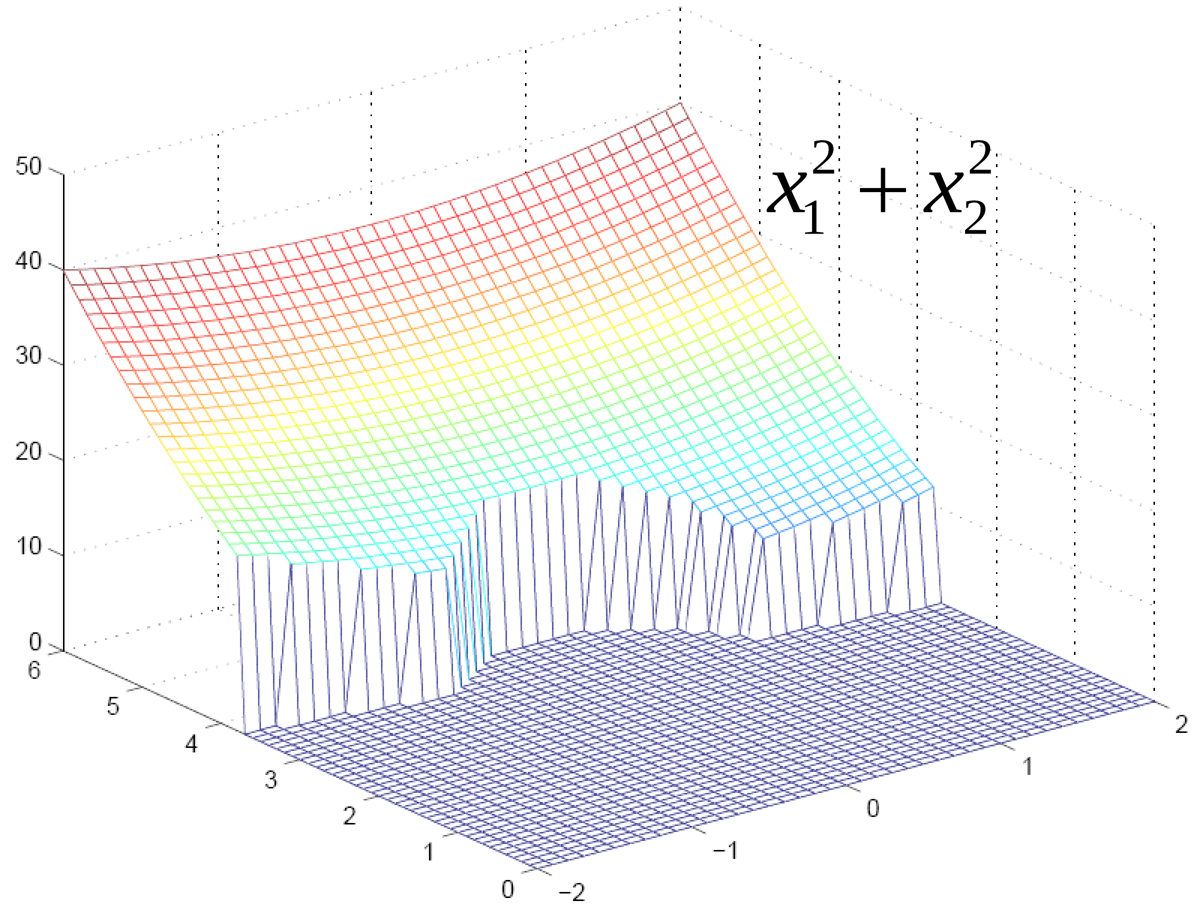
Example:
Nonlinear constrained optimization



Example:
Nonlinear constrained optimization



Example:
Nonlinear constrained optimization



Example:

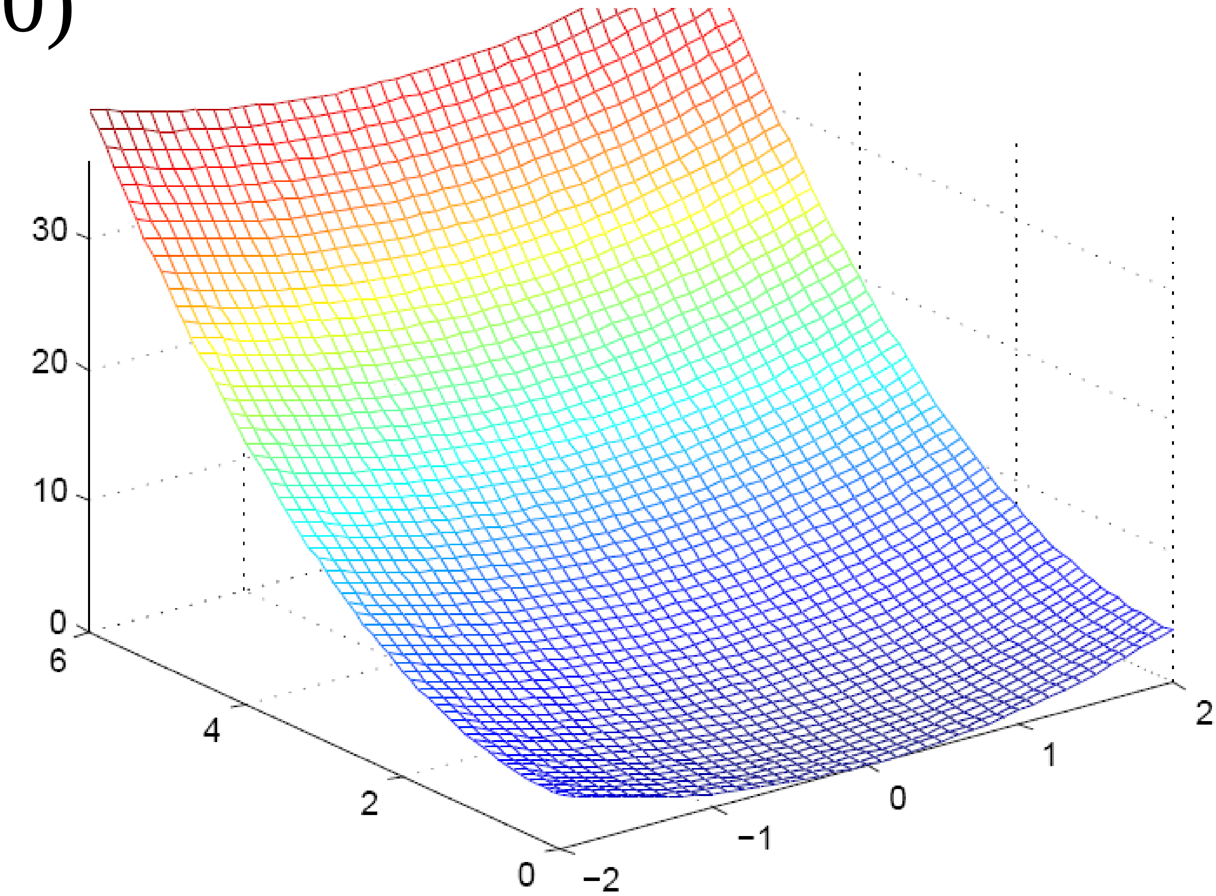
Nonlinear constrained optimization

$$
\begin{aligned}
\min \quad & x_1^2 + x_2^2 & \\
s.t. \quad & x_1^2 + x_2 & \geq 4 \\
& x_1 + 4x_2 & \geq 13
\end{aligned}
$$

Example:

$$L_y(x) = f(x) - y_1 g_1(x) - y_2 g_2(x)$$

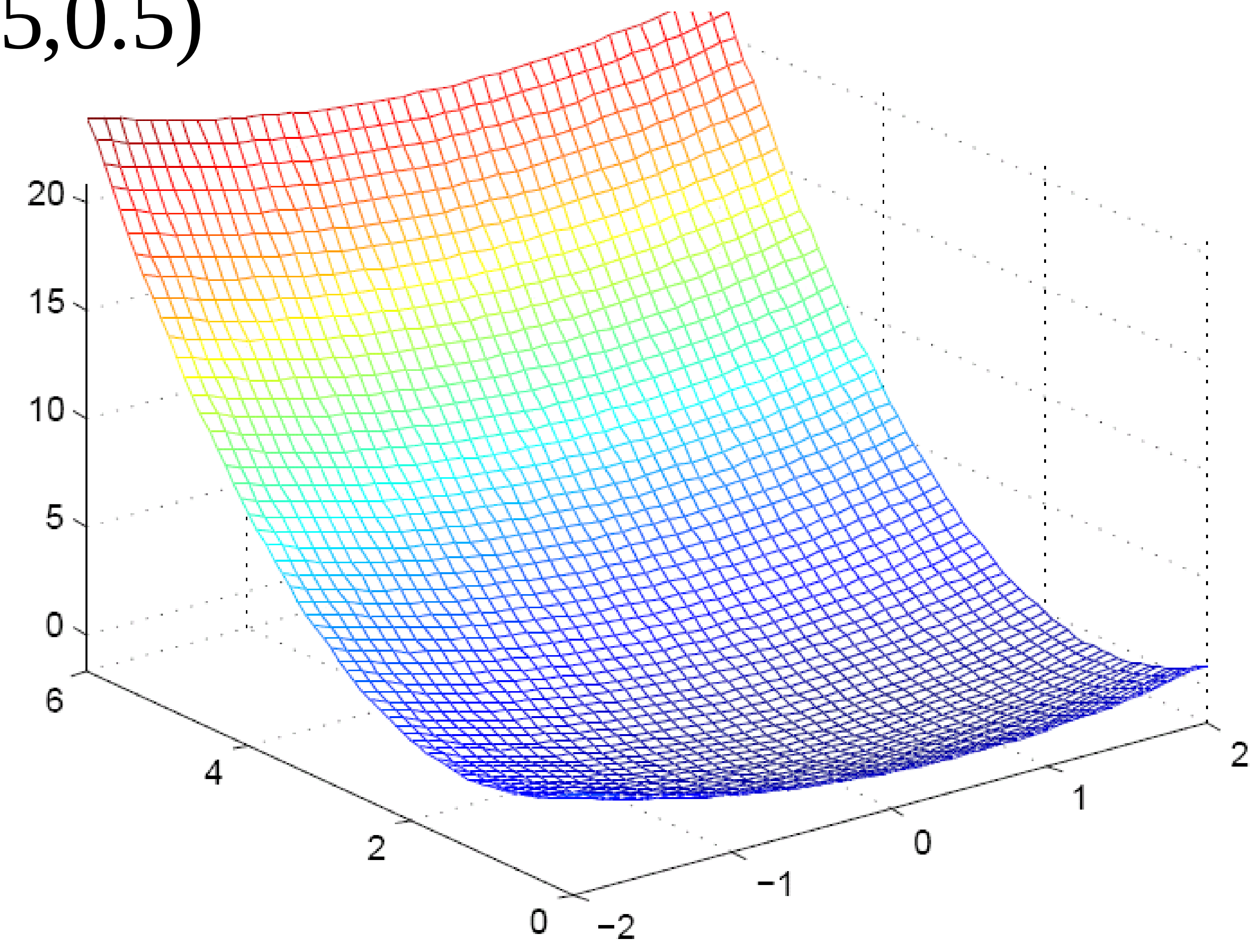
$$y = (0,0)$$



Example:

$$L_y(x) = f(x) - y_1 g_1(x) - y_2 g_2(x)$$

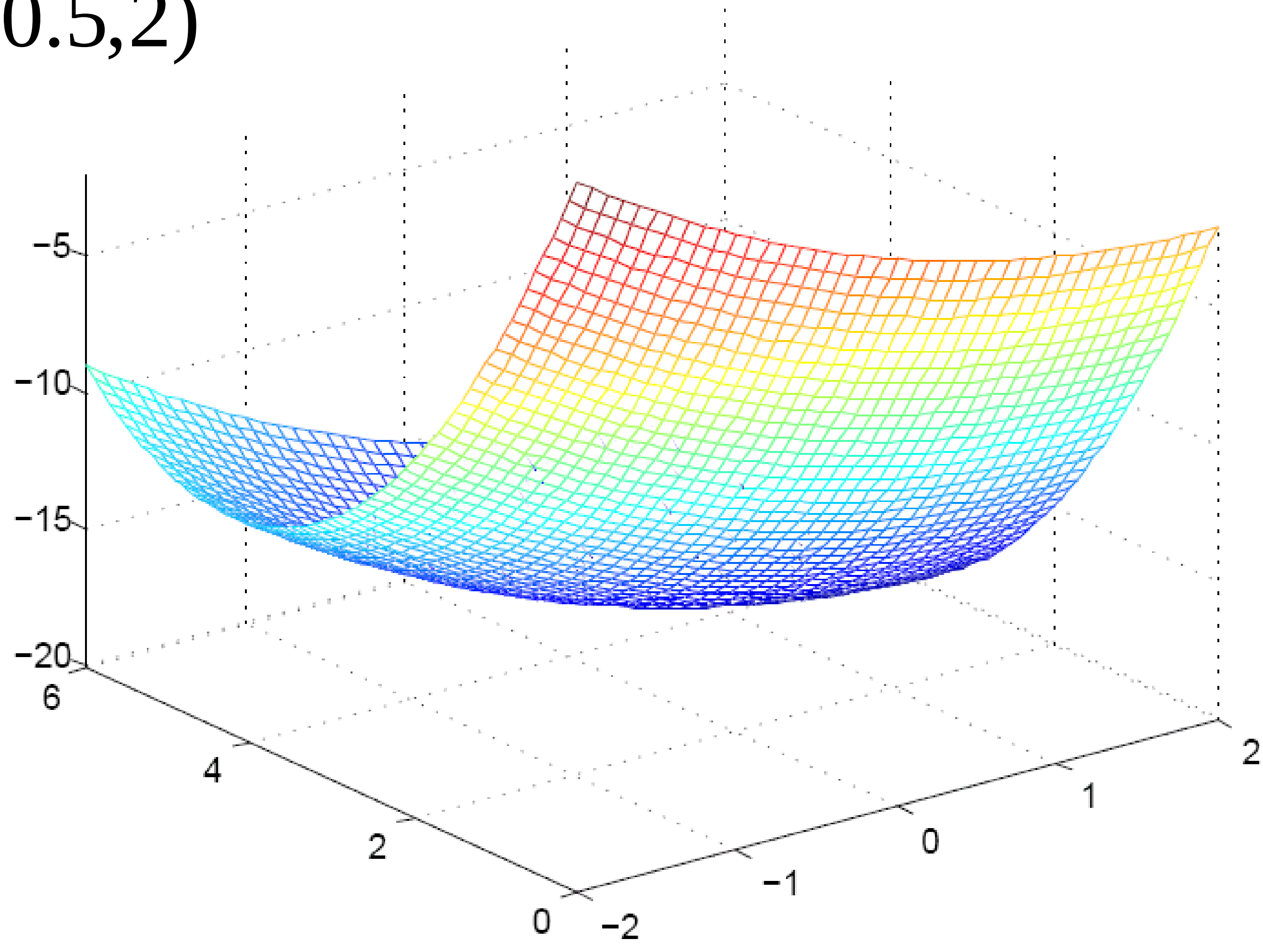
$$y = (0.5, 0.5)$$



Example:

$$L_y(x) = f(x) - y_1 g_1(x) - y_2 g_2(x)$$

$$y = (0.5, 2)$$



1. Unconstrained optimization

<u>Necessary condition of optimality</u>

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$$\nabla f(\bar{x}) = 0.$$

2. Constrained Optimization

$$
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\begin{array}{llll}
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s.t. & g_i(x) & \geq 0 & \forall i \in \{1,\ldots,p\} \quad (NLP) \\
 & h_j(x) & = 0 & \forall j \in \{1,\ldots,q\}
\end{array}
}
$$

i>clicker

Q: Which solution is optimal for our NLP example?

A. $x^A = (2, 0)$

B. $x^B = (10, 20.25)$

C. $x^C = (1, 3)$

D. $x^B = (10, 20.25)$ and $x^C = (1, 3)$

E. None of the above

Q: Which solution is optimal for our NLP example?

A. $x = (2, 0)$

B. $x = (10, 20.25)$

C. $x = (1, 3)$

D. $x = (10, 20.25)$ and $x = (1, 3)$

E. None of the above

AMPL