Lecture 6

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Previously in Opt 2 …

The maximum-flow and the minimum-cut problems

Maxflow

- Input
  - $G = (N, E)$, a directed graph
  - The node set N contains:
    - A source node, s
    - A sink node, t
  - u_{ij} = edge capacity; $u_{ij} \geq 0$
- Objective:
  To maximize net flow into t, subject to constraints:
  - Capacity constraints
  - Flow conservation constraints:

    Net flow out of node i = 0

    (for each node i in N, except s and t)

Mincut

- Input
  - $G = (N, E)$, a directed graph
  - The node set N contains:
    - A source node, s
    - A sink node, t
  - u_{ij} = edge capacity; $u_{ij} \geq 0$
- Objective: To find a cut with minimum cut capacity.
  - A cut: A partition of N into sets (S, T), where S contains the source and T contains the sink
  - Capacity of the cut (S, T) is

    the sum of capacities of edges that go from S to T:

$$\sum_{\substack{(i,j)\in E,\text{s.t.} \\ i\in S,\, j\in T}} u_{ij}$$

The Ford-Fulkerson method

0. Find an initial feasible solution (flow)
1. Consider the current solution, x.
   Construct a residual graph, G_x.
2. Using the labeling algorithm, try to find an augmenting path in G_x. Then,
   If there is an augmenting path,
       then use this path to improve the flow.
   If there is no augmenting path,
       then x is optimal. We're done! (A cut whose capacity is equal to the flow value is found.)

Finding an augmenting path (Finding nodes reachable from s)

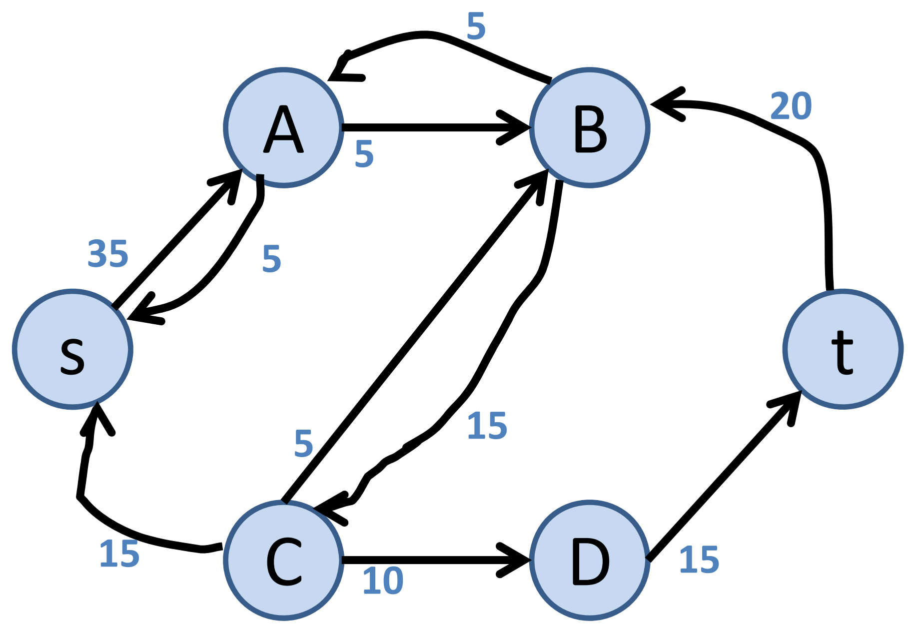
A labeling algorithm:

List of “checked” nodes: L = {s}

While L is not empty:

- Take the first node off the list L, suppose it’s node i.
- For each edge (i, j) out of i, if j is unchecked, check it and add j to the list L. Highlight the edge.
- (repeat)

Residual graph for $x^{(2)}$, $G_{x(2)}$



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- (repeat)

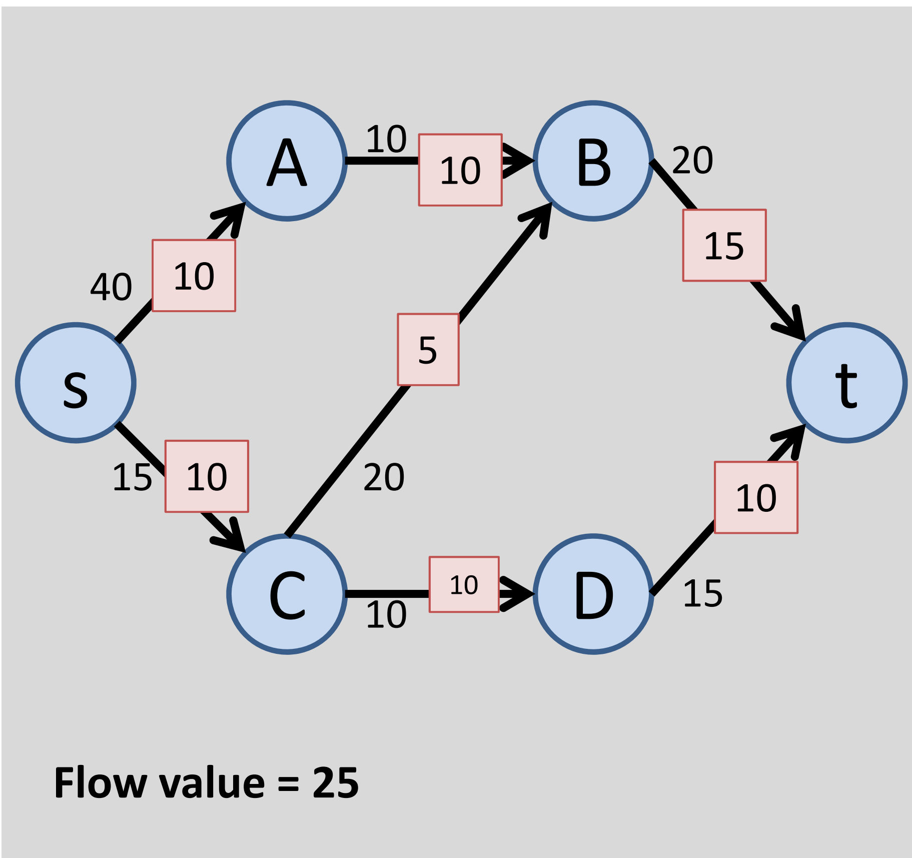
At the end of the algorithm one of the following happens:

1. The algorithm stops, and node t is not “checked”.
   Then, …
2. The algorithm stops, and node t is “checked”.
   Then, …

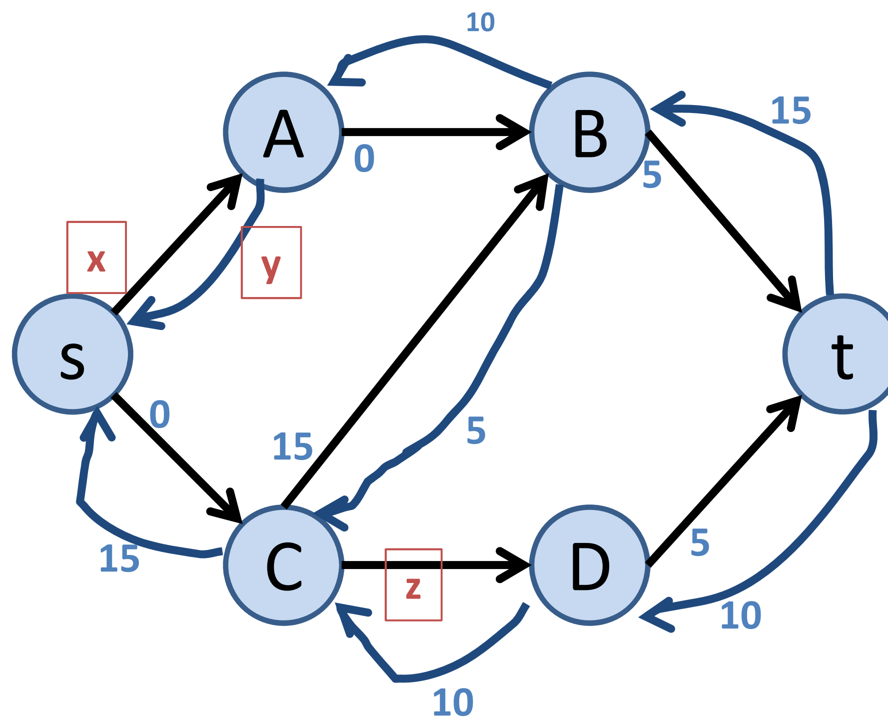
Q1 (i>clicker)

Q1:
Complete the residual graph for $x^{(k)}$

Current solution, $x^{(k)}$

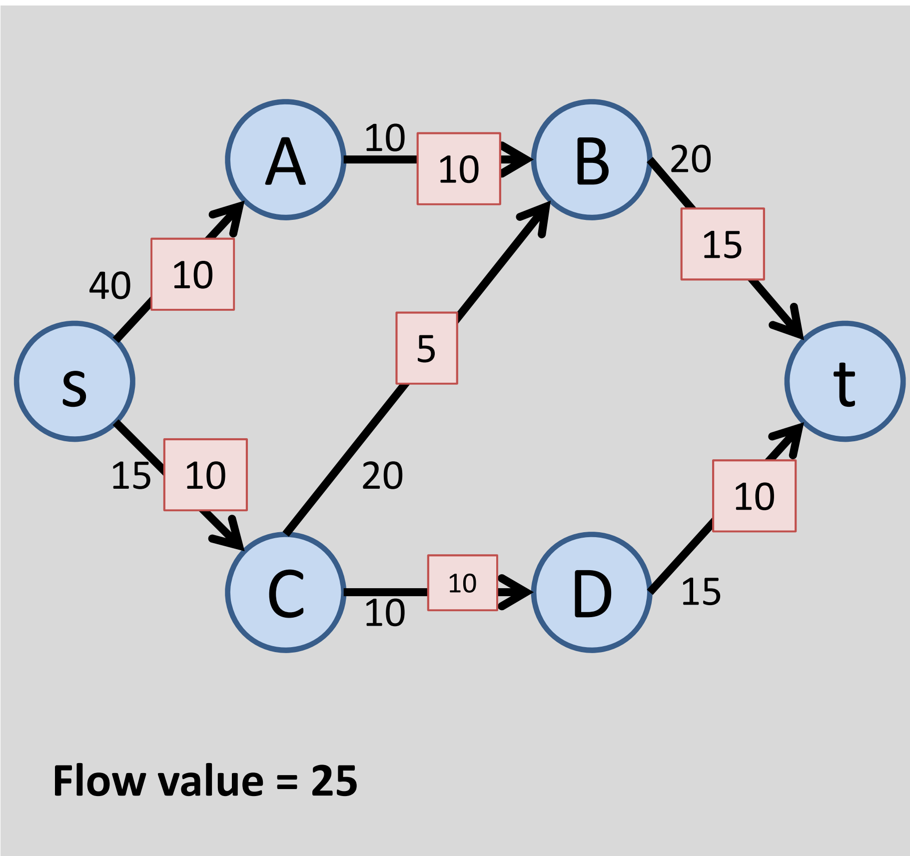


Residual graph for $x^{(k)}$, $G_{x(k)}$



Q1:
Complete the residual graph for $x^{(k)}$

Current solution, $x^{(k)}$



Residual graph for $x^{(k)}$, $G_{x(k)}$