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A formulation and theory for delay guarantees in wireless

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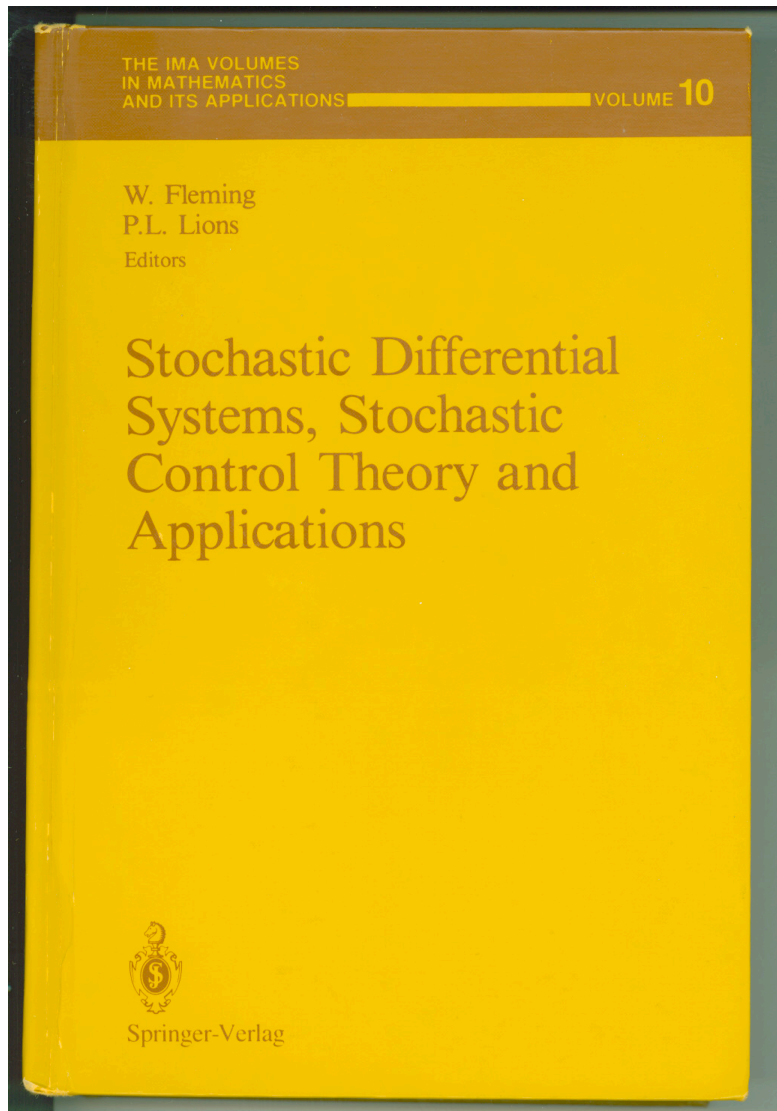
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Minneapolis, June 1986



BROWNIAN MODELS OF QUEUEING NETWORKS
WITH HETEROGENEOUS CUSTOMER POPULATIONS

by

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Consider an open queueing network with I single-server stations and K customer classes. Each customer class requires service at a specified station, and customers change class after service in a Markovian fashion. (With K allowed to be arbitrary, this routing structure is almost perfectly general.) There is a renewal input process and general service time distribution for each class. The correspondence between customer classes and service stations is in general many to one, and the service discipline (or scheduling protocol) at each station is left as a matter for dynamic decision making.

Assuming that the total load imposed on each station is approximately equal to its capacity, we show how to approximate the queueing system by a Brownian network, a type of crude but relatively tractable stochastic system model. The data of the approximating Brownian model are calculated in terms of the queueing system's parameters, including both the first and second moments of the interarrival and service time distributions. (The Brownian approximation is insensitive to specific distributional forms.) We do not attempt a rigorous convergence proof to justify the proposed approximation, but the argument given in support of the approximation amounts to a broad outline for such a proof.

The Brownian approximation is initially developed for a network of reliable single-server stations, then generalized to allow server breakdown, and finally extended to closed networks and networks with controllable inputs. In all cases examined thus far, the approximating Brownian network is more tractable than the conventional model it replaces, whether the objective is performance evaluation or system optimization.

1. Introduction

In a future paper [8] a class of crude but relatively tractable stochastic system models, called Brownian networks, will be systematically discussed. The three notions that are taken as primitive in such models are resources (indexed by $i=1, \dots, I$), activities (indexed by $j=1, \dots, J$) and stocks (indexed by $k=1, \dots, K$). The system dynamics and policy constraints of a Brownian network are compactly expressed by the following relations:

(1) $Z(t) = X(t) + RY(t) \in S$ for all $t \geq 0$, and

(2) $U(t) = AY(t)$ is a non-decreasing process with $U(0) = 0$.

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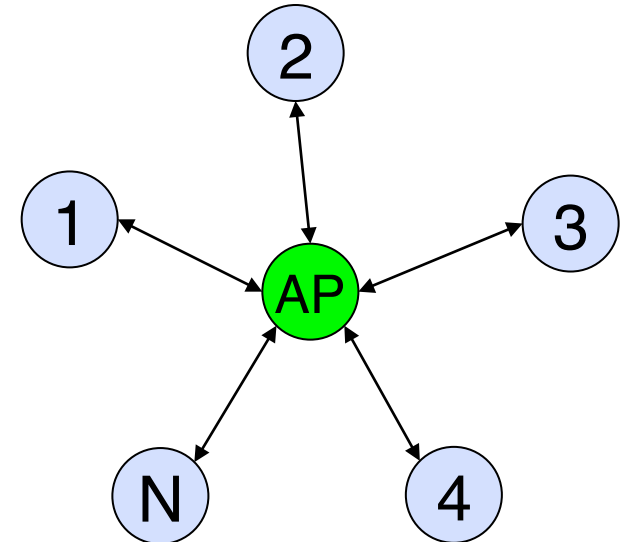
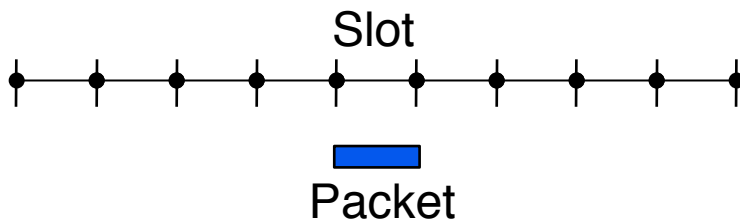
Challenge of providing delay guarantees for wireless

- ◆ Increasing use of wireless networks for serving traffic with delay constraints:
 - VoIP
 - Interactive Video
 - Networked Control
- ◆ Yet delay guarantees are not supported
- ◆ How to formulate a mathematical framework for delay-based QoS?
- ◆ Relevant: Jointly deal with several QoS issues
 - » Deadlines
 - » Delivery ratios
 - » Channel unreliabilities
- ◆ Tractable: Provide solutions for QoS support
 - » Admission control policies for flows
 - » Packet Scheduling policies



Client-Server model

- ◆ A wireless system with an Access Point serving N clients
- ◆ Time is slotted
- ◆ One slot = One packet



- ◆ AP indicates which client should transmit in each time slot



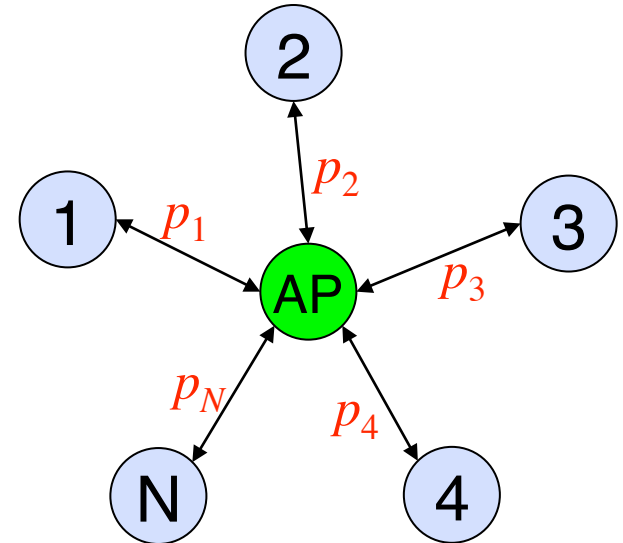
Model of unreliable channels

◆ Unreliable channels

◆ Packet transmission in each slot

- Successful with probability p_n
- Fails with probability $1-p_n$

- So packet delivery time is a geometrically distributed random variable γ_n with mean $1/p_n$

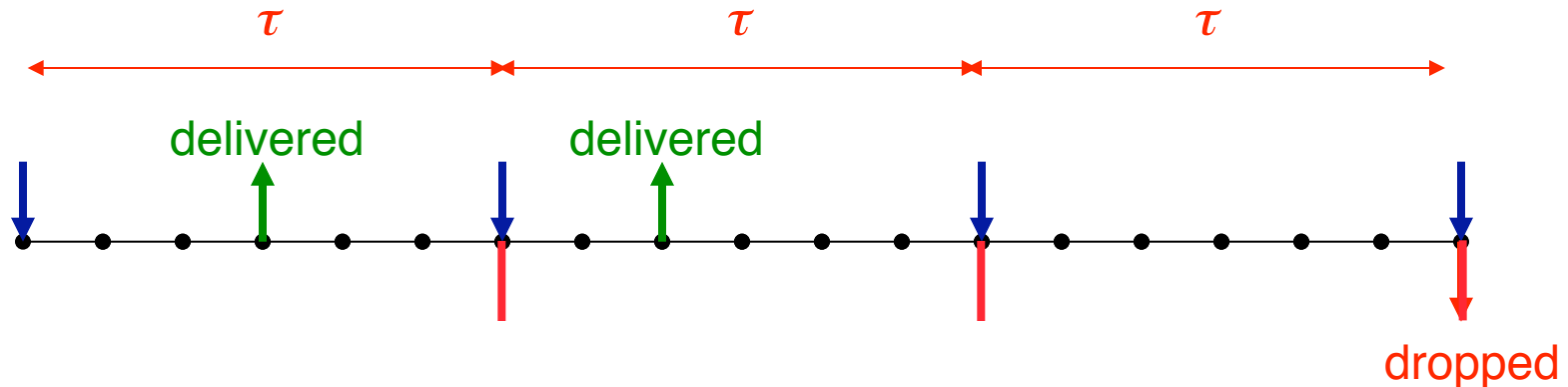


◆ Non-homogeneous link qualities

- p_1, p_2, \dots, p_N can be different



QoS model



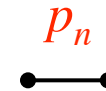
- ◆ Clients generate packets with fixed period τ
- ◆ Packets expire and are dropped if not delivered in the period
- ◆ Delay of successfully delivered packet is therefore at most τ
- ◆ Delivery ratio of Client n should be at least q_n

$$\liminf_{T \rightarrow \infty} \frac{1}{T} \sum_{t=1}^T 1(\text{Packet delivered to Client } n \text{ in } t\text{-th period}) \geq q_n \quad a.s.$$

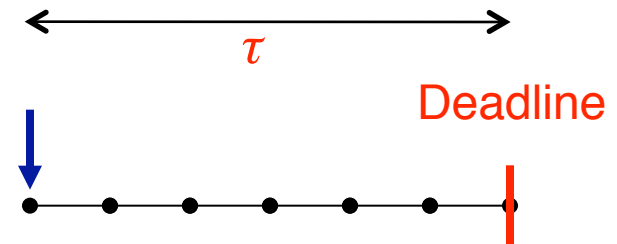


Multiple-time scale QoS requirements

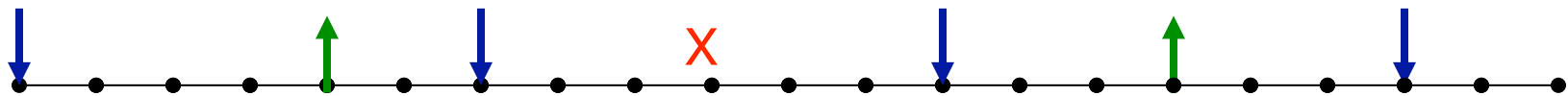
- ◆ Unreliable channels
 - Short time scale: Slots



- ◆ Arrivals and Deadlines
 - Medium time scale:
 - Period τ arrivals
 - Relative Deadline τ



- ◆ Delivery ratio requirements
 - Long time scale:

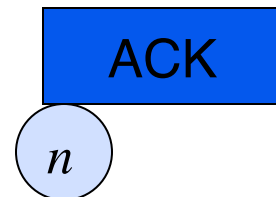
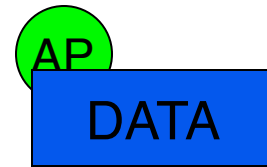


$$\liminf_{T \rightarrow \infty} \frac{1}{T} \sum_{t=1}^T 1(\text{Packet of client } n \text{ delivered in } t\text{-th period}) \geq q_n \text{ a.s.}$$



Protocol for operation

- ◆ AP indicates which client should transmit in each time slot
- ◆ Downlink
 - DATA
 - ACK
 - $p_n = \text{Prob}(\text{Both DATA and ACK are delivered})$



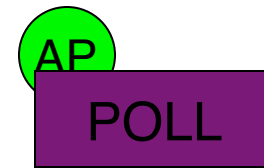


Protocol for operation

- ◆ AP indicates which client should transmit in each time slot

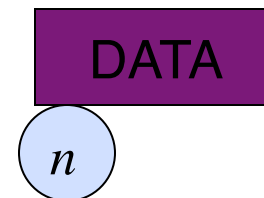
- ◆ Downlink

- DATA
- ACK
- $p_n = \text{Prob}(\text{Both DATA and ACK are delivered})$



- ◆ Uplink

- POLL (e.g., CF-POLL in 802.11 PCF)
- DATA
- $p_n = \text{Prob}(\text{Both POLL and DATA are delivered})$
- No need for ACK





Feasibility of a set of clients



Implied workload

- ◆ Workload due to Client n

$$w_n = \frac{E(\# \text{ deliveries per period}) \cdot E(\# \text{ slots per delivery})}{\# \text{ of slots of per period}}$$

$$= \frac{q_n \cdot \frac{1}{p_n}}{\tau}$$

- ◆ The proportion of time slots needed by Client n is

$$w_n = \frac{q_n}{p_n \tau}$$

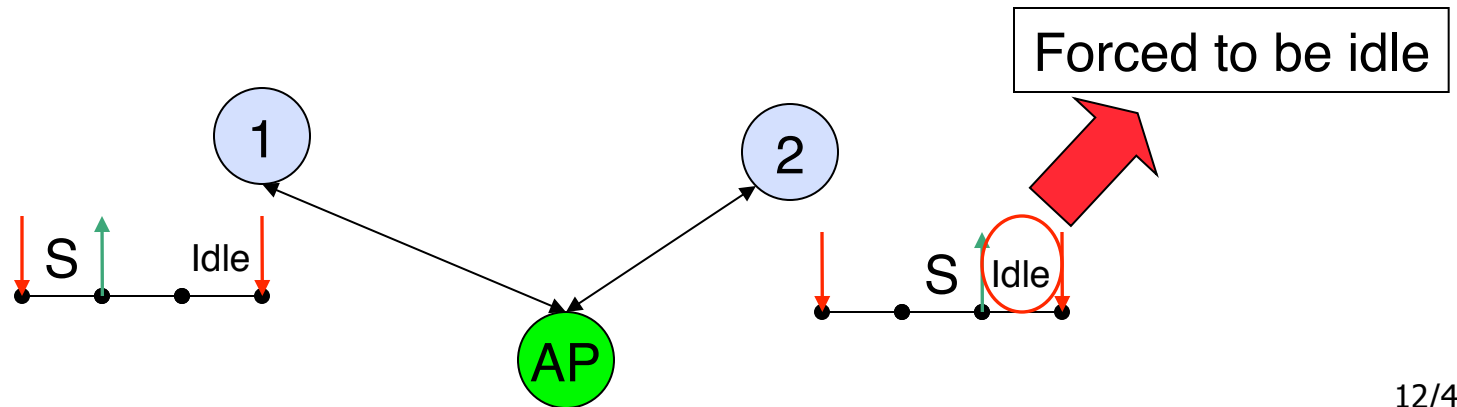


Necessary condition for feasibility of QoS requirements

- ◆ Necessary condition from classical queueing theory

$$\sum_{n=1}^N w_n \leq 1$$

- ◆ But not sufficient
- ◆ Reason: Unavoidable idle time
 - No queueing: At most one packet





Stronger necessary condition

- ◆ Let $I(1, 2, \dots, N) :=$ Unavoidable idle time after serving $\{1, 2, \dots, N\}$

$$I(1, 2, \dots, N) = \frac{1}{\tau} E \left[\left(\tau - \sum_{n=1}^N \gamma_n \right)^+ \right] \text{ where } \gamma_n \sim \text{Geom}(p_n)$$

- ◆ Stronger necessary condition

$$\sum_{n=1}^N w_n + I(1, 2, \dots, N) \leq 1$$

- ◆ Sufficient?
- ◆ **Still not sufficient!**



Counterexample

- ◆ Two clients: Period $\tau = 3$

- ◆ Client 1

- $p_1 = 0.5$
- $q_1 = 0.876$

- $w_1 + I_1 = 3.002/3 > 1$ ✘

$$w_1 = \frac{q_1}{p_1 \tau} = \frac{1.752}{3}$$

$$I_1 = \frac{(2p_1 + (1 - p_1)p_1)}{3} = \frac{1.25}{3}$$

- ◆ Client 2

- $p_2 = 0.5$
- $q_2 = 0.45$

- $w_2 + I_2 = 2.15/3 < 1$ ✔

$$w_2 = \frac{q_2}{p_2 \tau} = \frac{0.9}{3}$$

$$I_2 = \frac{1.25}{3}$$

- ◆ Clients {1,2}

- $w_1 + w_2 + I_{\{1,2\}} = 2.902/3 < 1$ ✔

$$w_{\{1,2\}} = w_1 + w_2 = \frac{2.652}{3}$$

$$I_{\{1,2\}} = \frac{p_1 p_2}{3} = \frac{0.25}{3}$$



Even stronger necessary condition

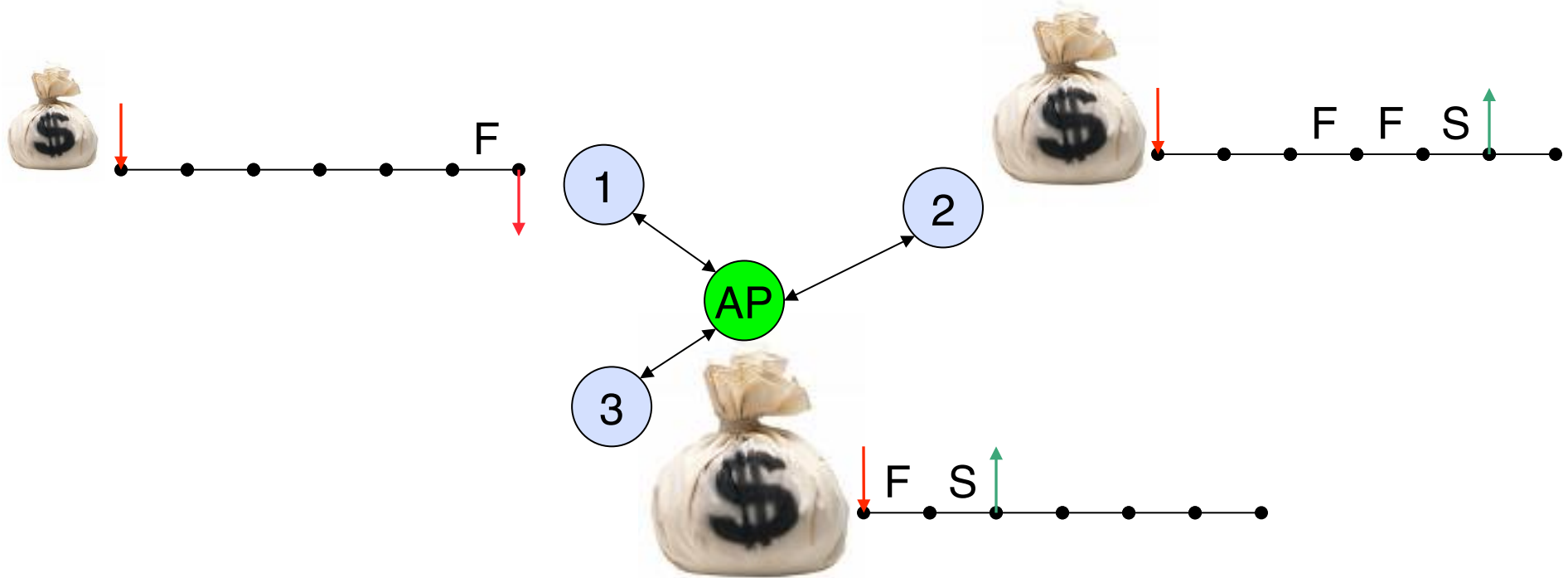
- ◆ Every *subset* of clients $S \subseteq \{1, 2, \dots, N\}$ should also be feasible
- ◆ Let $I(S) := \frac{1}{\tau} E \left[\left(\tau - \sum_{n \in S} \gamma_n \right)^+ \right]$ = Idle time if only serving S
- ◆ Stronger necessary condition: $\sum_{n \in S} w_n + I(S) \leq 1, \forall S \subseteq \{1, 2, \dots, N\}$
- ◆ Not enough to just evaluate for the whole set $\{1, 2, \dots, N\}$
- ◆ **Theorem (Hou, Borkar & K '09)**
Condition is necessary and sufficient for a set of clients to be feasible



Scheduling policy



Debt-based scheduling policies



- ◆ Compute “debt” owed to each client at beginning of period
- ◆ A client with higher debt gets a higher priority on that period



Two definitions of debt

- ◆ The **time debt** of Client n

w_n – Actual proportion of transmission slots given to Client n

- ◆ The **weighted delivery debt** of Client n

$\frac{q_n - \text{Actual delivery ratio of client } n}{P_n}$

- ◆ **Theorem (Hou, Borkar & K '09)**

Both largest debt first policies fulfill every set of clients that can be fulfilled



Proof



Blackwell's theory of approachability

- ◆ Period t
- ◆ Action $u(t)$
- ◆ Reward *vector* $r(t) \in R^N$
 - Distribution of $r(t)$ depends only on $u(t)$
 - Mean reward = $E(r \mid u)$
- ◆ Let $\rho(T) := \frac{1}{T} \sum_{t=1}^T r(t)$ = Time average of rewards up to stage T
- ◆ Consider a set $A \subseteq R^N$

- ◆ **Definition of an approachable set A**

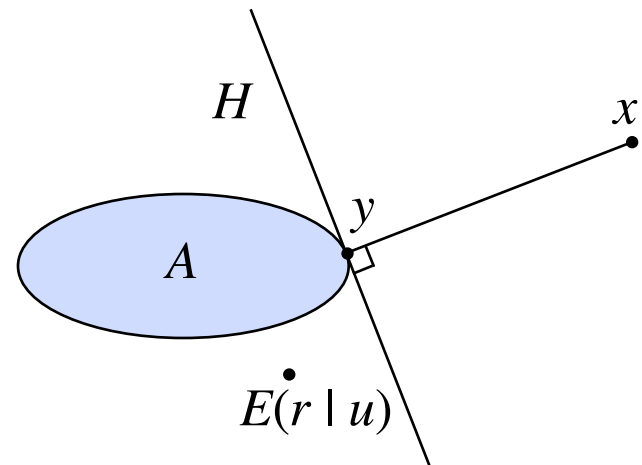
For some policy, for every $\varepsilon > 0$ and $\delta > 0$, there is a T_0 such that

$$P(\text{Dist}(\rho(T), A) < \varepsilon \text{ for all } T \geq T_0) > 1 - \delta$$



Blackwell's sufficient condition for approachability

- ◆ **Theorem (Blackwell '56)**
- ◆ Suppose A is closed
- ◆ Suppose for every $x \notin A$, there is an action u such that the mean reward $E(r | u)$ lies on the other side of the hyperplane H passing through y , the point in A closest to x , and perpendicular to the line xy
- ◆ Then A is approachable under this policy, where any action can be taken when $x \in A$.

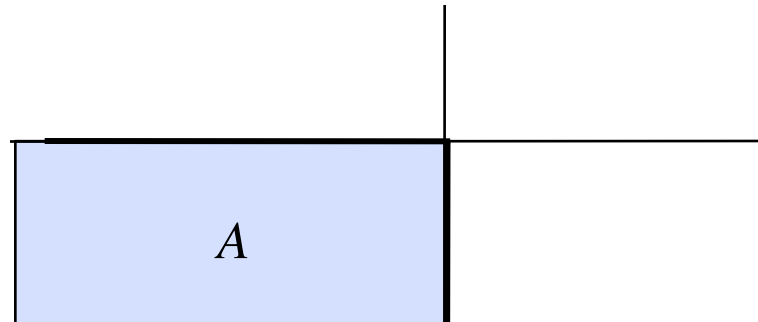




Proof that time-debt policy is feasibility optimal

- ◆ Period t
- ◆ Action $u =$ Priority determined by *time-debt policy*
- ◆ $r_n = n$ -th component of resulting reward
 - $:= \tau w_n$ - Time spent on Client n in period
- ◆ Time average of rewards up to stage $T =$ Time-debt
 - Want all debts non-positive

- ◆ $A =$ Non-positive orthant of R^N



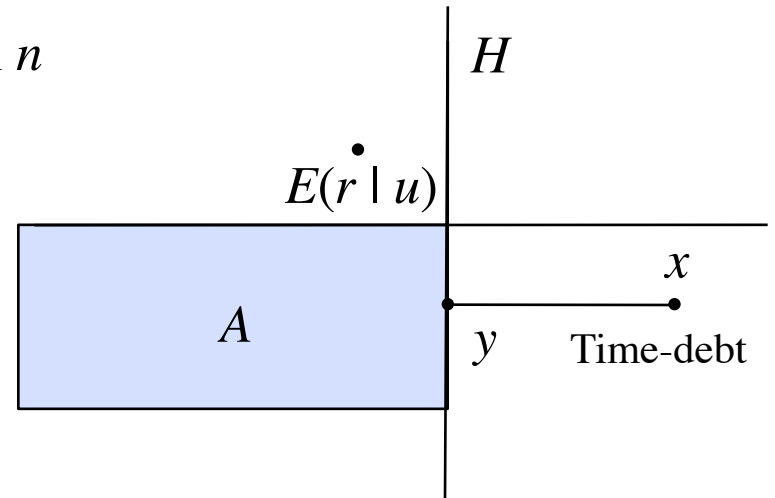


Sufficient condition for approachability of A

◆ Order $x = (\underbrace{x_1, x_2, \dots, x_m}_{>0}, \underbrace{x_{m+1}, \dots, x_N}_{\leq 0})$ with $x_n \searrow$ in n

◆ Then $y = (0, 0, \dots, 0, x_{m+1}, \dots, x_N)$

◆ Hyperplane is $H := \{z : (z - y)^T (x - y) = 0\}$
 $= \{z : \sum_{n=1}^m z_n x_n = 0\}$



◆ Now x is in the positive half-space of H

◆ So we only need to show that $\sum_{n=1}^m E(r_n | u) x_n \leq 0$

◆ I.e., we only need to show $\sum_{n=1}^m E(\tau w_n - \underbrace{\text{Time spent on Client } n \text{ in period}}_{:= B_n}) \cdot x_n \leq 0$



Approachability proof

- ◆ Let $B_n =$ Busy time spent on Client n
- ◆ Priority order for service is $1, 2, 3, \dots, k, \dots, m, m+1, \dots, N$
- ◆ So for $1 \leq k \leq m$, $\tau - \sum_{n=1}^k E(B_n) \stackrel{\text{Priority for } \{1,2,\dots,k\}}{=} \tau I(1,2,\dots,k) \stackrel{\text{Feasibility}}{\leq} \tau - \tau \sum_{n=1}^k w_n$
- ◆ So $\alpha^k := \sum_{n=1}^k (\tau w_n - E(B_n)) \leq 0$ for all $1 \leq k \leq m$
- ◆ Then
$$\begin{aligned} \sum_{n=1}^m E(\tau w_n - B_n) \cdot x_n &= \sum_{n=1}^m (\alpha^n - \alpha^{n-1}) x_n \\ &\leq \sum_{n=1}^m (\alpha^n x_n - \alpha^{n-1} x_{n-1}) \quad (\text{Since } x_n \searrow \text{ in } n). \\ &= \alpha^m x_m \quad (\text{Set } \alpha^0 := 0 \text{ and } x_0 := x_1). \\ &\leq 0. \quad (x_k > 0 \text{ for } 1 \leq k \leq m). \end{aligned}$$



Computationally tractable admission control



Computationally tractable policy for admission control

- ◆ Admission control consists of determining feasibility
- ◆ We need to check: $\sum_{n \in S} w_n + I_S \leq 1, \forall S \subseteq \{1, 2, \dots, N\}$
- ◆ Apparently 2^N tests, so computationally complex, but
- ◆ **Theorem (Hou, Borkar & K '09)**
 - Order the clients according to q_n in decreasing order
 - Then $\{1, 2, \dots, N\}$ infeasible $\Leftrightarrow \sum_{n=1}^k w_n + I(1, 2, \dots, k) > 1$ for some k
 - So we need only N tests to check $\{1, 2, \dots, k\}$ for $1 \leq k \leq N$
 - Polynomial time $O(N\tau \log \tau)$ algorithm for admission control



Proof of polynomial time test

- ◆ Say a set S is *bad* if $\sum_{n \in S} w_n + I(S) > 1$
- ◆ Suppose $S = \{ \overset{1}{\bullet} \overset{2}{\bullet} \overset{3}{\bullet} \overset{4}{\circ} \overset{5}{\bullet} \circ \circ \bullet \bullet \circ \bullet \bullet \overset{j}{\circ} \bullet \bullet \overset{m}{\bullet} \circ \circ \circ \circ \circ \circ \overset{N}{\circ} \}$ is *bad*
- ◆ But $S-m = \{ \overset{1}{\bullet} \overset{2}{\bullet} \overset{3}{\bullet} \overset{4}{\circ} \overset{5}{\bullet} \circ \circ \bullet \bullet \circ \bullet \bullet \overset{j}{\circ} \bullet \bullet \overset{m}{\circ} \circ \circ \circ \circ \circ \circ \overset{N}{\circ} \}$ *not bad*
- ◆ Will show $S' := S+j = \{ \overset{1}{\bullet} \overset{2}{\bullet} \overset{3}{\bullet} \overset{4}{\circ} \overset{5}{\bullet} \circ \circ \bullet \bullet \circ \bullet \bullet \overset{j}{\bullet} \bullet \bullet \overset{m}{\bullet} \circ \circ \circ \circ \circ \circ \overset{N}{\circ} \}$ is *bad*
 - S' has one less hole than S
- ◆ Now prune $m, m-1, \dots, m-n$ till we get a set that is *not bad*
- ◆ Repeat with $S'-m-(m-1)-\dots-(m-n+1)$



Proof of polynomial time test

- ◆ Give highest priority to $S-m$. Next priority to m . Last priority to j .

- ◆ Then
$$\sum_{n \in S+j} w_n + I(S+j) = \left(\sum_{n \in S} w_n + I(S) \right) + w_j - \frac{E(B_j)}{\tau}$$

- ◆ So to show $S+j$ is bad, it is sufficient to show
$$w_j - \frac{E(B_j)}{\tau} \geq 0$$

- ◆ Now
$$w_j - \frac{E(B_j)}{\tau} = w_j - \frac{1}{\tau} \sum_{\sigma=1}^{\tau} P(m \text{ completed with } \sigma \text{ slots left}) \cdot E(B_j | \sigma \text{ slots left})$$

$$\geq w_j - \frac{1}{\tau} \sum_{\sigma=1}^{\tau} P(m \text{ completed with } \sigma \text{ slots left}) \cdot E(B_j | \infty \text{ slots left})$$

$$\geq w_j - \frac{1}{\tau p_j} \sum_{\sigma=1}^{\tau} P(m \text{ completed with } \sigma \text{ slots left})$$

$$\stackrel{S \text{ is bad}}{\geq} w_j - \frac{1}{\tau p_j} q_m = \frac{1}{\tau p_j} q_j - \frac{1}{\tau p_j} q_m \stackrel{q_j \geq q_m}{\geq} 0.$$



Complexity of admission control algorithm

- ◆ Order q_1, q_2, \dots, q_N in decreasing order
- ◆ Evaluate w_1, w_2, \dots, w_N , where $w_n = \frac{q_n}{\tau p_n}$
- ◆ Evaluate $g_m(t) := \text{Prob}(\text{Packets of } 1, 2, \dots, m \text{ are delivered by } t)$ by FFT

$$g_m(t) = \sum_{s=1}^{\tau} (1 - p_m)^{s-1} p_m g_{m-1}(t - s)$$

- ◆ Evaluate $I(1, 2, \dots, m) = \frac{1}{\tau} \sum_{s \geq 1} P(\text{Number of idle slots} \geq s) = \frac{1}{\tau} \sum_{s \geq 1} g_m(\tau - s)$

- ◆ So $O(N\tau \log \tau + N \log N)$



Proof that weighted delivery debt policy is feasibility optimal

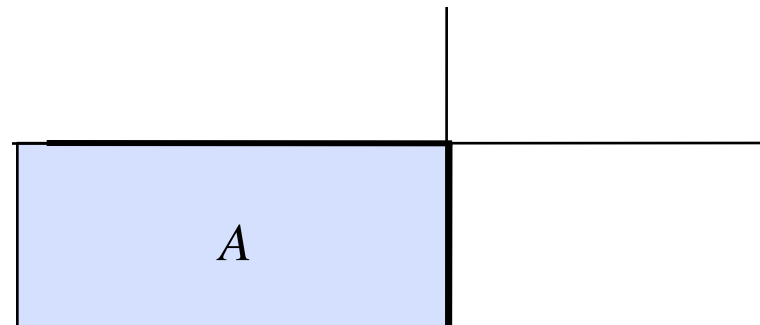
- ◆ Period t
- ◆ Action $u =$ Priority determined by *weighted delivery debt policy*

- ◆ $r_n = n$ -th component of resulting reward

$$:= \frac{q_n}{p_n} - \frac{1(n \text{ is successfully delivered})}{p_n}$$

- ◆ Time average of rewards up to stage $T =$ Weighted delivery debt

- ◆ $A =$ Non-positive orthant of R^N



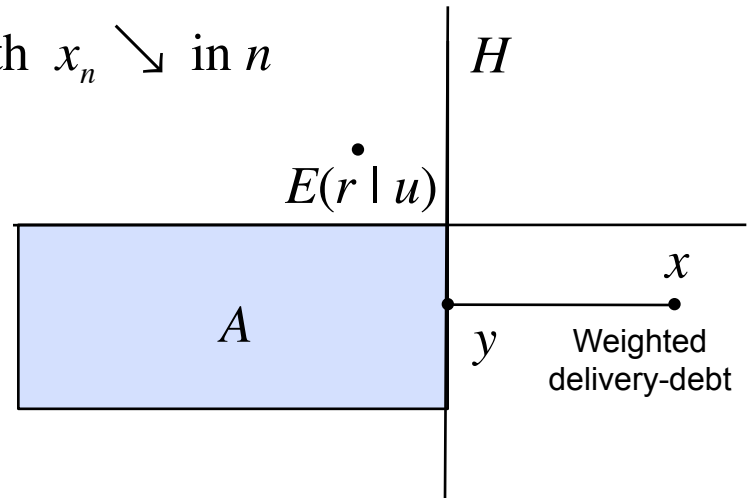


Sufficient condition for approachability of A for weighted delivery debt policy

- ◆ As before order $x = (\underbrace{x_1, x_2, \dots, x_m}_{>0}, \underbrace{x_{m+1}, \dots, x_N}_{\leq 0})$ with $x_n \searrow$ in n

- ◆ Now we need to show

$$\sum_{n=1}^m E\left(\frac{q_n - 1(n \text{ is successfully delivered})}{p_n}\right) \cdot x_n \leq 0$$



- ◆ If $\pi_n := P(n \text{ is successfully delivered})$

- ◆ We need to show $\sum_{n=1}^m \left(\frac{q_n - \pi_n}{p_n}\right) \cdot x_n \leq 0$

- ◆ As before it suffices to show $\sum_{n=1}^k \frac{q_n - \pi_n}{p_n} \leq 0$ for $1 \leq k \leq m$



Approachability proof for weighted-delivery debt policy

- ◆ Note $\frac{\pi_1}{p_1} = \frac{1 - (1 - p_1)^\tau}{p_1} = 1 + (1 - p_1) + \dots + (1 - p_1)^{\tau-1} = P(B_1 \geq 1) + \dots + P(B_1 \geq \tau)$
 $= E(B_1)$

- ◆ Similarly, conditioned on $\{1, 2, \dots, k-1\}$ completing σ slots before end of period

$$\frac{P(k \text{ is successful} | \sigma \text{ slots left when } \{1, 2, \dots, k-1\} \text{ completes})}{p_k} = \frac{1 - (1 - p_1)^\sigma}{p_1}$$

$$= 1 + (1 - p_1) + \dots + (1 - p_1)^{\sigma-1} = E(B_k | \sigma \text{ slots left to serve } k)$$

- ◆ So $\frac{\pi_k}{p_k} = E(B_k)$

- ◆ Hence
$$\sum_{n=1}^k \frac{q_n - \pi_n}{p_n} = \sum_{n=1}^k \tau w_n - E(\text{Busy time serving } \{1, 2, \dots, k\})$$

$$= \sum_{n=1}^k \tau w_n - (\tau - I(1, 2, \dots, k)) \leq 0$$



Main results

◆ Theorem (Hou, Borkar & K '09)

– A set of clients $\{1, 2, \dots, N\}$ is feasible



– The weighted delivery debt policy satisfies all the clients

} Scheduling policy



$$\sum_{n=1}^k \frac{q_n}{p_n \tau} + \frac{1}{\tau} E \left[\left(\tau - \sum_{n=1}^k \gamma_n \right)^+ \right] \leq 1 \quad \text{for all } k = 1, 2, \dots, N$$

} Feasibility characterization

– An $O(N\tau \log \tau + N \log N)$ Admission Control Policy

} Admission control policy



Simulation testing



Simulation testing on ns-2

- ◆ Implement on IEEE 802.11 Point Coordination Function (PCF)
 - Point Coordinator (PC) assigns transmission opportunities to clients
 - Packets should be sent by broadcasting to avoid ACKs
 - Compatible with Distributed Coordination Function (DCF)

- ◆ Application: VoIP standard

64 kbp data rate	20 ms period
160 Byte packet	11 Mb/s transmission rate
610 μ s time slot	32 time slots in a period

- ◆ Four policies
 - DCF and PCF with randomly assigned priorities
 - **Time-debt policy** and **Weighted-delivery debt policy**

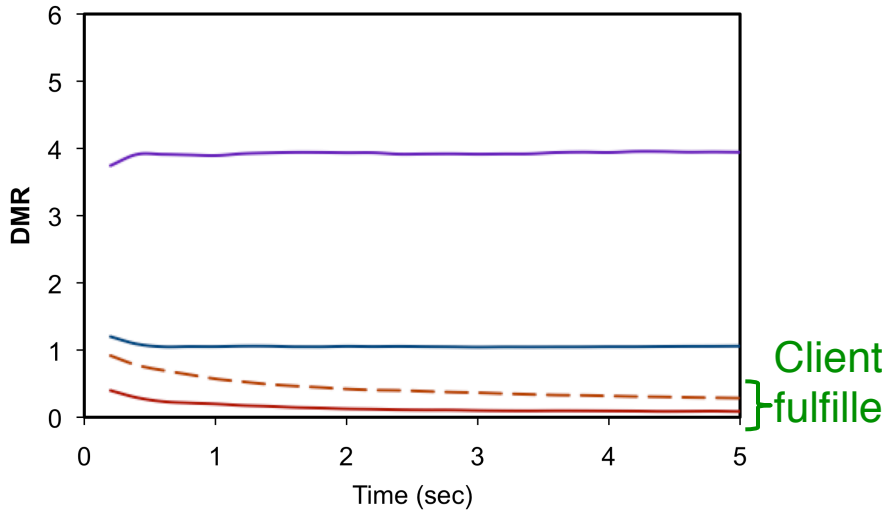


Traffic requirements: Test at edge of feasibility

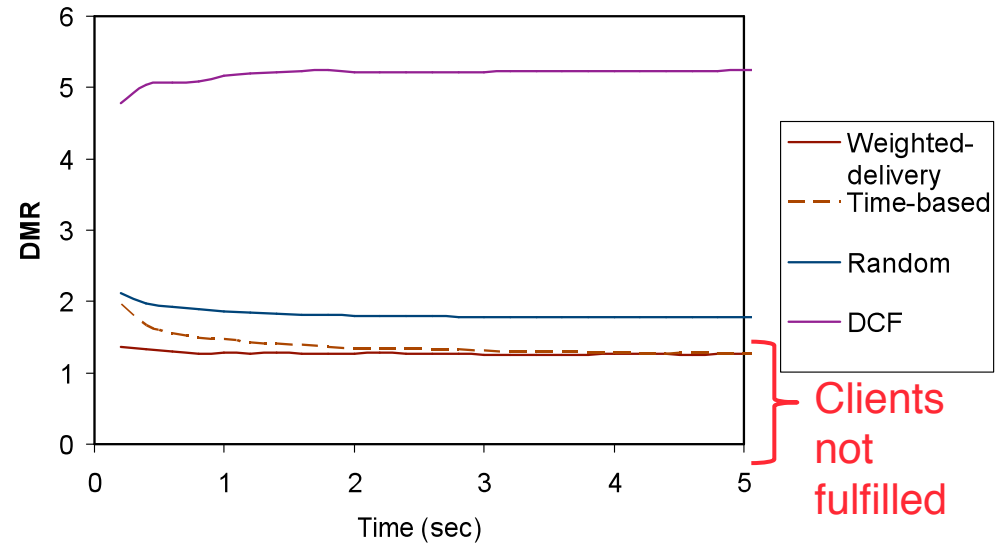
- ◆ Two groups of clients
 - Group A requires 99% delivery ratio
 - Group B requires 80% delivery ratio
 - The n^{th} client in each group has $(60+n)\%$ channel reliability
- ◆ Feasible set: 11 group A clients and 12 group B clients
- ◆ Infeasible set: 12 group A clients and 12 group B clients
- ◆ Evaluation Measure
 - $DMR(n) := (q_n - \text{percentage of actual delivered packets})^+$
- ◆ DMR of system = $\sum_{n=1}^N DMR(n)$



Results



Feasible set



Infeasible set



Extensions ...



More general arrivals

◆ Theorem (Hou & K '09)

- Suppose $r(S)$ = Probability that packets for set S arrive in a period
 - » Packet need not arrive in every period. (Can extend to periodic arrivals)
 - » Client arrivals can be correlated

- Then, a set of clients $\{1, 2, \dots, N\}$ is feasible

$$\sum_{n \in S} \frac{q_n}{p_n} + \sum_{G \subseteq \{1, 2, \dots, N\}} r(G) E \left[\left(\tau - \sum_{n \in S \cap G} \gamma_n \right)^+ \right] \leq \tau \quad \text{for all } S \subseteq \{1, 2, \dots, N\}$$

- The weighted delivery debt policy satisfies all the clients



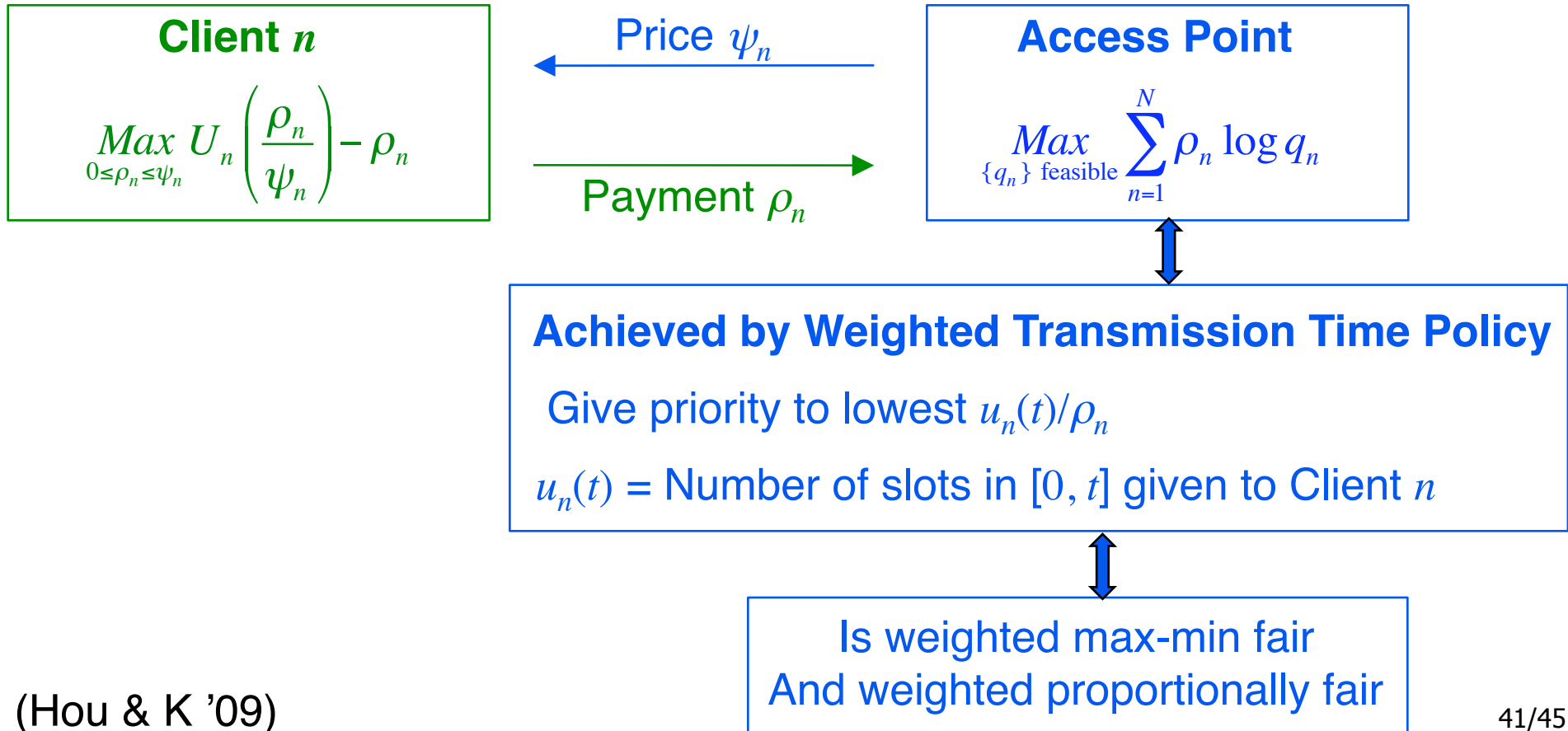
Time varying channels, heterogeneous deadlines, rate adaptation, etc

- ◆ More general packet arrivals at beginning of periods
- ◆ Clients with different deadlines
- ◆ Either
 - No rate adaptation and unreliable channel, or
 - Rate adaptation with reliability
- ◆ Time varying channels
- ◆ Pseudo-debt $r_n(t)$: Clients fulfilled $\Leftrightarrow \lim_{t \rightarrow \infty} \frac{r_n(t)}{t} \leq 0$
- ◆ **Theorem (Hou & K '09)**
 - Let μ_n = Expected reduction in pseudo-debt for Client n
 - μ_n depends on the scheduling policy
 - Policy that maximizes $\sum_n \mu_n r_n^+(t)$ is feasibility optimal.



Utility maximization

- ◆ **System** $Max_{\{q_n\} \text{ feasible}} \sum_{n=1}^N U_n(q_n)$ (U_n str. concave, str. Incr., $U_n(0) = \text{right limit}$)





Conclusion

- ◆ A framework for delay-based QoS that deals with
 - deadlines
 - delivery ratios
 - channel unreliabilities
 - fading channels
 - general arrivals
 - rate adaptation
 - client utilities, etc

- ◆ Analytically tractable

- ◆ Implementable policies for admission control and scheduling



References

- ◆ I-Hong Hou, V. Borkar and P. R. Kumar, “A Theory of QoS for Wireless.” *Proceedings of Infocom 2009*, April 19-25, 2009, Rio de Janeiro, Brazil.
- ◆ I-Hong Hou and P. R. Kumar, “Admission Control and Scheduling for QoS Guarantees for Variable- Bit-Rate Applications on Wireless Channels.” *Proceedings of MobiHoc 2009*, pp. 175–184. New Orleans, May 18-21, 2009.
- ◆ I-Hong Hou and P. R. Kumar, “Utility Maximization for Delay Constrained QoS in Wireless.” (<http://arxiv.org/abs/0908.0362v1>). July 31, 2009.
- ◆ I-Hong Hou and P. R. Kumar, “Scheduling Heterogeneous Real-Time Traffic over Fading Wireless Channels” (<http://arxiv.org/abs/0908.0587v1>). July 31, 2009.



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